Effect of hydrostatic pressure on a bubble anechoic metascreen

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Abstract – Bubble metascreens consist of a single layer of gas inclusions in an elastomer. They can be used as ultra-thin coatings for turning acoustic reflectors into perfect absorbers. The effectiveness of such a coating at a chosen frequency is mainly determined by three parameters: the size of the bubbles, the distance between them, and the rheology of the elastomer. If any of these parameters vary during the use of the coating, the performance is affected. We used numerical simulations to investigate the effect of the static pressure on the acoustic properties of bubble metascreens with spherical or cylindrical inclusions.

I. INTRODUCTION

Viscoelastic materials with periodic distribution of holes, sometimes called “Alberich tiles”, have been studied for several decades for their subwavelength acoustic behavior, which makes them good candidates for use as anechoic coatings on underwater vehicles [1–6]. In a recent work [7], we proposed an analytical model for the transmission and reflection coefficients of a single layer of bubbles in a soft solid. We showed that the acoustic absorption of the metascreen could be maximized, at a given frequency, by adapting the geometry of the layer (radius of the spherical bubbles and distance between them) to the viscosity of the elastomer.

The good low-frequency performance of the metascreen rests on the compressibility of the inclusions, i.e. on the low shear modulus of the elastomer. A consequence is that the structure is easily deformed when submitted to a static pressure, which impacts the acoustic performance of the metascreen. This is a major limitation of the technique for underwater applications. Not many studies in the literature are devoted to the effect of the static pressure, or the temperature, on the acoustic properties of perforated elastomers. Gaunaurd et al carried out a theoretical analysis of the problem [8]. More recent studies found that pressure, and especially temperature played an important role on the global performance of the metascreen [9].

In the present study, we use numerical simulations to study the nonlinear deformation of an elastomer with gas inclusions, on a rigid backing, submitted to static pressures. Spherical and cylindrical inclusions are considered. Then dynamic simulations are performed to determine the reflection coefficient of the metascreen under different levels of compression.

II. DESCRIPTION OF THE SYSTEM

We ran numerical simulations with Comsol Multiphysics. The response of an infinite array of cavities in an elastomer was obtained by considering a single cell, as sketched in Fig. 1. We note $e$ the thickness of the elastomer, and $d$ its lateral length. A rigid condition was imposed on the lateral faces of the system, and the same condition on the bottom face was used to mimic the presence of an infinitely rigid support.

Fig. 1: Geometry of the metascreen considered in the study.
A cavity was placed in the middle of the layer, with a spherical (radius $A$) or a cylindrical (diameter $D$, height $H$) shape. The physical parameters of the elastomer were the following: density $\rho = 1100$ kg/m$^3$, bulk modulus $K = 2$ GPa and shear modulus $G_0 = 1.5$ MPa. With these values, the reflection at the water-elastomer interface was negligible (phase velocity $v = 1.35$ mm/$\mu$s, acoustic impedance $Z = 1.48$ MRayl). The geometry of the metascreen was chosen in such a way that the reflection coefficient was minimal at 2 kHz. We obtained good performance (see Fig. 3b) for $e = 50$ mm ($\lambda/15$ in water), $d = 185$ mm and $A = 9$ mm.

### III. Compression Tests

We simulated an uniaxial compression caused by a hydrostatic pressure $P_{\text{ext}} = P_0 + \Delta P$ that pushes the metascreen against its rigid backing. The initial volume of the cavity is noted $V_0$, and its volume after compression $V$. Here we consider a void cavity but the presence of air can be taken into account and was checked not to qualitatively change our results. Fig. 2 reports the change of volume obtained ($V/V_0$) as a function of the extra pressure $\Delta P$ for three kinds of inclusion: a sphere (blue points), a cylinder with an aspect ratio (AR) of 1 ($D = H$), and a flatter cylinder ($D = 4H$). The nonlinear response of the material was considered, with a neo-hookean hyperelasticity. Note that the maximum extra pressure considered here was enough to obtain a deviation from the linear Gaunaurd law. Extrapolating from a classical result for thin spherical elastic shells [10], we propose the following relationship between the extra-pressure and the relative change of volume $x = V/V_0$:

$$\Delta P = 2G_0 \left( x^{-1/3} - y^{-1/3} + \left( x^{-4/3} - y^{-4/3} \right) / 4 \right),$$

where $y = 1 + \Phi(x - 1)$, $\Phi$ being the volume fraction of air in the elastomer ($\Phi = V_0/(\pi D^2)$). Note that this equation is obtained for a uniform compression. However, as shown in Fig. 2, it is in excellent agreement with the simulation results. Visualisation of the deformation (inset of Fig. 2) confirms that the deformed inclusion remains close to a sphere. The same simulations were carried out with the two cylinders, both with the same volume as the sphere (left), the cylinder (middle) and the flat cylinder (right). The simulations could be run with different static compressions, giving access to the effect of the metascreen deformation on its reflection coefficient.

### IV. Effect of Compression on Acoustic Behavior

The same configuration can be used to simulate the acoustical response of the metascreen. A layer of water was added on top of the elastomer, with a perfectly matched layer (PML) to simulate an infinite medium. To account for the frequency-dependence of the shear modulus, we used a fractional Zener model: $G(f) = [G_0 + G_1(\omega \tau_0)^\alpha]/[1 + (\omega \tau_0)^\beta]$ with $G_1 = 100$ MPa, $\tau = 3.5$ $\mu$s and $\alpha = 0.6$ ($\omega = 2\pi f$ is the angular frequency and $i^2 = -1$). $\alpha$ is the shift factor using the Williams-Landel-Ferry (WLF) model : $lg \omega_T = -C_1 (T - T_{\text{ref}}) / (c_2 + T - T_{\text{ref}})$ with $c_1 = 10$, $c_2 = 100$ and the reference temperature $T_{\text{ref}} = 0^\circ C$; the working temperature is $T = 20^\circ C$. The simulations could be run with different static compressions, giving access to the effect of the metascreen deformation on its reflection coefficient.

Fig. 3a shows the results for the sphere. Before compression, a deep minimum of reflection (around $-30$ dB) is obtained at 2 kHz. The analytical model we proposed in a previous article [7] is found to be in reasonable agreement with the simulation (solid line). When a 7 bars extra-pressure is applied, the performance of the metascreen...
degrades. Qualitatively, it can be understood by the fact that the geometry of the array of cavities is not optimised any more, because the size of the spheres has changed. The analytical model predicts the loss of performance due to this change of geometry.

For the cylinders, the minimum of reflection is shifted to a lower frequency compared to the case of a sphere (see Fig. 3b), which is consistent with the higher compressibility of cylinders exhibited in Fig. 2. Interestingly, despite their higher loss of volume under compression, cylinders do not exhibit a larger degradation of their acoustic performance. In particular, for the flat cylinder, there is no clear frequency shift of the minimum of reflection when the extra-pressure goes from 0 to 7 bars. An hypothesis to explain this observation is that the acoustic resonance of the cylinders is dominated by the vibration of their upper and lower surfaces, whose diameter is not much affected by the compression (see insets in Fig. 2).

![Graphs showing reflection coefficients for spheres and cylinders under compression.](image)

(a) Results for spherical inclusions. (b) Numerical results for the two cylindrical inclusions.

Fig. 3: Analytical (solid lines) and numerical (symbols) analysis of the reflection coefficient on the metascreen placed in front of a perfect reflector, before and after compression at 7 bars.

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**REFERENCES**