Subharmonic generation of edge wave over a plane beach by breaking waves

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Outline
1. Introduction
2. Theoretical model for edge wave excitation
3. Scheem of experiment and results.
4. Discussion of results.
5. Conclusions.
1. Introduction. Cusp formation.
There exists theory for cusp generation: edge wave are responsible for formation of this pattern.

Standing edge model gives $\lambda_c = \lambda_{\text{edge}}/2$, $\lambda_{\text{edge}}$ is the edge wave.

The most important question is the following: how the edge waves in the ocean are excited?

Edge waves may appear as a result of parametric excitation.


Regular parametric forcing

How wave breaking influences the parametric excitation?
2. Theoretical model for edge wave excitation

\[ \eta = F(x) \cos(\omega t - kx) \]
Shallow water approximation 
\[ \eta = b \cos(\Omega_n t - ky) \cdot e^{-kx} L_n(x) \]

\[ F_n(x) = e^{-kx} L_n(x) \]

\( L_n \) are polynomials of Laguerre.

\[ \Omega_n = \sqrt{(2n+1) \beta g k} \]
Standing edge waves $n=0$

$$\eta = 2b \cos(\Omega_1 t) \sin(ky) \cdot e^{-kx}$$
\[ \eta = a_0 J_0 \left( \sqrt{\frac{4 \omega^3 x}{g \tan \beta}} \right) \cos(\omega t) \]
Equation for slowly varying amplitude of edge waves

Whitham, JFM, 1976, Akylas, JFM, 1983, ...

\[ \frac{\partial b}{\partial t} = -\gamma b + h b^* + i\Delta b + (i\sigma - \rho)b|b|^2 \]

\( \gamma \) is for decay constant

\( \Delta = \Omega - \omega/2 \) is for detuning

\[ h = a_0 \frac{\omega^3}{4g\beta^2} S(\beta) \] is amplitude of parametric forcing

Linear approximation: threshold of parametric excitation

\[ h = \sqrt{\Delta^2 + \gamma^2} \]

P1-P3 are probes.
Breaking parameter $Br$ 

$$Br = \frac{\max(\frac{dU}{dt})}{gt\tan\beta}$$

$Br \approx 0.9$

$Br \approx 1.4$

$0.9 < Br < 1.7$
channel width $D=0.5\text{m}$

edge wave wave number \[ k = \frac{\pi}{D} \]
Development of parametric instability

run-up before instability                               parametric instability

Frequency of excitation $f=1.06$ Hz
Plane frequency- run-up amplitude

$R_{\text{max}}$ [cm]

$\text{inviscid theory}$  $\text{viscid theory}$
Wave breaking regime, run-up before parametric excitation of edge waves $f=1.06$ Hz, $R_{\text{max}}=2.8$ cm.
Wave breaking regime, run-up corresponding parametric excitation of edge waves. $f=1.06$ Hz, $R_{\text{max}}=3\text{cm}$. 
Dependence of edge wave amplitude exponential growth rate on amplitude of run-up due to surface waves
Dependence of edge wave amplitude on amplitude of run-up due to surface waves
4. Discussion of results

a. Curves of marginal stability


Region of parametric instability is shifted to the lower frequencies

Our experiments
Dependences of run-up amplitude on amplitude of external forcing, \( f=1.06 \text{ Hz} \)
b. Suppression of parametric instability

Φ phase of parametric pumping is non regular


\[
\frac{\partial b}{\partial t} = -\gamma b + h b^* e^{-\langle \Phi^2 \rangle / 2} + i\Delta b + (i\sigma - \rho)b|b|^2
\]
Hilbert transformation

\[ \sqrt{\langle \Phi^2 \rangle} \approx 0.11 \]

\[ e^{-\left(\frac{\langle \Phi^2 \rangle}{2}\right)} \approx 0.995 \]
Fluctuations of phase cannot explain the suppression of parametric instability.

Threshold of parametric instability

\[ \gamma = h \]

For \( f = 1.06 \) Hz parametric instability exists for

\[ 0.8 \text{cm} < R_{\text{max}} < 3.1 \text{cm} \]

Increasing of run-up in 4 times leads to the increasing of turbulent viscosity and coefficient of edge wave decay more than 4 times. It leads to the suppression of parametric instability.
5. Conclusions

1. Parametric instability of edge waves increases maximal run-up.
2. Wave breaking leads to suppression of parametric instability of edge waves.
3. Suppression of instability is due to increasing of turbulent viscosity.