Dissipation-driven interaction between non-propagating solitons and boundaries

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Non-linear Hydrodynamic Waves, Paris 2013
Overview: Faraday waves

vertical vibrations
- Two parameters:
  - Amplitude: $a_0$
  - Frequency: $\omega$

Spatiotemporal structure:
- Double period
- $\omega \uparrow$, $\lambda \downarrow$

energy injection \(\rightarrow\) transition \(\rightarrow\) dissipative system \(\rightarrow\) out-of-equilibrium

Faraday waves

1831

Vertical vibrations

Spatiotemporal structure:
Overview: Non-propagating hydrodynamic soliton

Same experiment but... in high aspect-ratio basin.

\[ b \ll 1 \]

\[ \nu_{1,0} = 1.14 \text{ [Hz]} \]
\[ \nu_{2,0} = 2.17 \text{ [Hz]} \]
\[ \nu_{3,0} = 3.05 \text{ [Hz]} \]
\[ \nu_{4,0} = 3.77 \text{ [Hz]} \]
Overview: Non-propagating hydrodynamic soliton

Same experiment but...
in high aspect-ratio basin.

$\nu_{0,1} = 5.50 \, [\text{Hz}]$

$\nu_{1,1} = 5.53 \, [\text{Hz}]$

$\nu_{2,1} = 5.60 \, [\text{Hz}]$

$\nu_{3,1} = 5.72 \, [\text{Hz}]$
Overview: Non-propagating hydrodynamic soliton

Same experiment but... in high aspect-ratio basin.
Overview: Non-propagating hydrodynamic soliton

Same experiment but... in high aspect-ratio basin.

driving at frequency corresponding to (0,1) surface mode + perturbation

\[ \nu_{0,1} < 5.50 \text{ [Hz]} \]

energy injection \xrightarrow{\text{transition}} \text{Dissipative system} \xrightarrow{\text{out-of-equilibrium}} \text{localized structure SOLITON}

\[ b \ll 1 \]

Observation of a Nonpropagating Hydrodynamic Soliton

Junru Wu, Robert Keolian, and Isadore Rudnick
Department of Physics, University of California, Los Angeles, California 90024
(Received 30 January 1984)
Theoretical framework
Hydrodynamic equations

Continuum mechanics

Hydrodynamics

Kinemetic condition:

\[ \partial_z \Phi = \partial_t \eta + \nabla_\perp \eta \cdot \nabla_\perp \Phi \]

Dynamic condition:

\[ \partial_t \Phi + \frac{1}{2} (\nabla \Phi)^2 + g \eta - \sigma \kappa = 0 \]

At \( z = \eta(x, y) \)

Mass

\( \nabla \cdot u = 0 \)

Incompressible

Laplace equation

\( \nabla^2 \Phi = 0 \)

Free surface at \( S \)

\( \nabla^2 \Phi = 0 \)

Bulk at \( V \)

\( u \cdot \hat{n} = 0 \)

Rigid walls at \( W \)

Bernoulli equation

Irrotational flow

Navier-Stokes

Inviscid

\( \mu = 0 \)
Variational principle

The hydrodynamic problem is complicated.

* Is it possible to reduce the problem to only 2D?

\[ \varepsilon = \int_{-d(x)}^{\eta(x,t)} \frac{1}{2} \rho (\nabla \Phi)^2 \, dz + \frac{1}{2} \rho g \eta^2 + \gamma \left( \sqrt{1 + (\nabla_\perp \eta)^2} - 1 \right) \]

Energy density (water column):

- Kinetic energy: flow
- Potential energy: gravity
- Elastic energy: surface tension

Action:

\[ I = \int_{0}^{t} \int_{R} \rho \left( \xi \partial_t \eta - \frac{\varepsilon}{\rho} \right) \, dS \, dt + I_0, \quad \xi = \Phi \big|_{z=\eta} \]

Variational principle and Hamilton equations:

\[ \mathcal{L} = \xi \partial_t \eta - \mathcal{H} \quad \implies \quad \partial_t \xi = -\frac{\delta \mathcal{H}}{\delta \eta}, \quad \partial_t \eta = \frac{\delta \mathcal{H}}{\delta \xi}. \]
pd-NLS equation

- Amplitude equation for (0,1) modulation $\psi \sim \eta + i\xi$

  - parametrical dissipative nonlinear Schrödinger equation

$$\partial_t \psi + \mu \psi = -i\nu \psi - iA |\psi|^2 \psi - iB \partial_{xx} \psi - i\gamma \overline{\psi}.$$  

+ boundary condition: $\partial_x \psi |_{x=\pm \frac{l}{2}} = 0$

- Parameters: $\tau = \tanh kd$

$$A = \frac{1}{64} k^2 \left( 6\tau^2 - 5 + 16\tau^{-2} - 9\tau^{-4} \right),$$

$$B = \frac{1}{4k^2} \left[ 1 + kd \left( \frac{1 - \tau^2}{\tau} \right) \right],$$

$$\gamma = \frac{\Gamma_0}{4g},$$

$$\nu = \frac{1}{2} \left[ \left( \frac{\omega}{\omega_1} \right)^2 - 1 \right].$$

$\mu$ = models should take them into account... (parametrical dissipative system)
Solutions of pd-NLS

- **Constant phase solutions** \( \psi(x, t) = \rho(x) \exp(-i\theta) \)

- **Solitons:**
  \[
  \sin(2\theta) = \gamma^{-1}
  \]
  \[
  \rho_\pm^s(x) = \delta_\pm \text{sech} \left[ \delta_\pm (x - x_0) \right]
  \]
  \[
  \delta_\pm^2 = -\nu \pm \sqrt{\gamma^2 - 1}
  \]

\[
\begin{align*}
\rho_s^+ (x) &= \delta_+ \text{sech} \left[ \delta_+ (x - x_0) \right] \\
\rho_s^- (x) &= \delta_- \text{sech} \left[ \delta_- (x - x_0) \right]
\end{align*}
\]

- Solutions appear by pairs (polarity)
Dynamics related to solitons

- Pair interaction depending on polarity
- Numerical approach (and some experiments)
  - Same phase -> attraction and bound states
  - Out of phase -> repulsion
  - Interaction with walls

\[ \gamma^2 = \nu^2 + \mu^2 \]

**Diagram**: Dynamical behavior of parametrically excited solitary waves in Faraday’s water trough experiment

Wei Wang, Xintong Wang, Junyi Wang, Rongjue Wei
Experiments
Image-based interface detection

Extra slabs mimic extra walls.

Detection based on adaptive threshold.

Spatiotemporal diagrams
Spatiotemporal diagrams
(no added slabs)

- Long measurements show weak soliton drift.
- Solitons can be created almost anywhere but...
- Solitons’ final position is independent from initial conditions.

Does this mean that walls repel solitons?

Equilibrium position depends on tilt angle of the trough.
Observation of wall-repulsion

- Boundary condition in pd-NLS model: \( \partial_x \psi \big|_{x=\pm \frac{1}{2}} = 0 \)
- Walls = image soliton configuration
- Solitons should form bounded states with walls
- Disagreement with results.
- More proofs:
  - Added slab between opposite-sign solitons

What are we missing in the model?

Let us analyze dissipation coefficient in pd-NLS equation.
Damping coefficient

- $\mu$ is essential for a good characterization.
- Physical origin? Back to hydrodynamics...
  - Effects of small but non-zero viscosity:
    - The flow is inviscid in the bulk.
    - Boundary layers at walls and free surface.
      - At dominant order, energy is dissipated in this region.
  - Capillary hysteresis.
    - Contact angle induces frictional force.
    - Extra dissipation term.

- Computation of values
  - For stationary waves
    - Adds effect of a viscoelastic film on the surface (surfactant).

Surface-wave damping in closed basins

By J. W. Miles
Institute of Geophysics and Planetary Physics,
University of California, La Jolla, California
Damping coefficient

• $\mu$ is essential for a good characterization.
  • Physical origin? Back to hydrodynamics...
    • Effects of small but non-zero viscosity:
      • The flow is inviscid in the bulk.
      • Boundary layers at walls and free surface.
        • At dominant order, energy is dissipated in this region.
    • Capillary hysteresis.
      • Contact angle induces frictional force.
      • Extra dissipation term.
  • Computation of values
    • For stationary waves
      • Adds effect of a viscoelastic film on the surface (surfactant).

\[
\alpha_W = \frac{\pi}{2} \left\{ \frac{1}{2} (J + K) + \frac{2}{\sinh 2kd} \left[ 1 - \frac{1}{2} kd (J - K) \right] \right\} \tanh (kd) \epsilon_\nu + O (\epsilon_\nu^2).
\]
pd-NLS equation with hands

\[ \partial_t \psi + \mu \psi = -i \nu \psi - iA |\psi|^2 \psi - iB \partial_{xx} \psi - i\gamma \overline{\psi}. \]

- Chain of nonlinear coupled oscillators.
- Coupling through dispersion term
- Each oscillator is characterized by:
  - natural frequency
  - forcing
  - cubic nonlinearity
  - damping

Boundary layers
- at \( x = \{0, l\} \)
- at \( y = \{0, b\} \)
- at \( z = -d \)

Lateral walls induce extra boundary layer!

All the parameters are constant along the chain except \( \mu \).
Damping coefficient in presence of walls

- Numerical simulations for three different set of parameters.
  - slab insertion protocol
  - (inset) only boundary conditions (b.c.).
  - (main) including b.c. + lateral wall correction

Singular distribution of dissipation coefficient induces wall-repulsion interaction.

Model reproduces correctly experimental results.

🌟 Drastic change of behavior at long time scales.
Conclusions
Soliton-wall interactions are driven by a dissipative mechanism that substantially alter the system dynamics at long time scales.

Damping in amplitude equations may be tricky:

- its spatial independence can be broken by the inherent system configuration (e.g. confinement)
- Common feature in dissipative out-of-equilibrium systems (e.g. dissipation mechanism).

Damping spatial dependance is impactful on localized solutions.

- Even highly localized subtle effects can absolutely modify the system final outcome.

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Dissipation-driven behavior of non-propagating hydrodynamic solitons under confinement

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Thanks!