Self-similar evolution of the Kelvin-Helmholtz instability

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A schoolyard near Lourdes
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Helmholtz & Kelvin
Origin of the instability

No excitation  Periodic excitation

Fig. 4.4.  Organ pipes. From Helmholtz [1877] p. 149.

Fig. 4.5.  Smoke jets, with (a) spontaneous instability, and (b) sound-triggered instability. From Becker and Massaro [1968]: plate 1. Courtesy of Henry Becker.
XLIII. *On Discontinuous Movements of Fluids.*

*By Professor Helmholtz.*

The stationary forms of the surfaces of division are distinguished, as experiment and theory alike indicate, by a remarkably high degree of alterability when subjected to the least disturbance, so that they comport themselves in some degree like bodies in unstable equilibrium. The remarkable sensitiveness to sound of a cylindrical current of air impregnated with smoke has already been described by Dr. Tyndall. I have verified Dr. Tyndall’s experiments. This is clearly a property of the surfaces of separation, which is of the greatest importance in sounding musical pipes.

Theory points out that, wherever an irregularity is formed on the surface of an otherwise stationary current, this must give rise to a progressive spiral unrolling of the corresponding part of the surface; the corresponding portion, moreover, advances along the current. This endeavour towards spiral unrolling at every interruption is also easily recognizable on observing the currents. According to theory, a prismatic or cylindrical current could be infinitely long. But in practice such a current cannot be formed, because in an element which is so easily moved as air it is impossible entirely to avoid small disturbances.
XLVI. Hydrokinetic solutions and observations
William Thomson

368 Sir W. Thomson on the Influence of Wind

Part III. The Influence of Wind on Waves in water supposed frictionless. (Letter to Professor Tait, of date August 16, 1871.)

Taking OX vertically downwards and OY horizontal, let

\[ x = h \sin n(y - \alpha t) \]  \hspace{1cm} (1)

be the equation of the section of the water by a plane perpendicular to the wave-ridges; and let \( h \) (the half wave-height) be infinitely small in comparison with \( \frac{2\pi}{n} \) (the wave-length). The \( x \)-component of the velocity of the water at the surface is then

\[ -nah \cos n(y - \alpha t); \hspace{1cm} (2) \]

and this (because \( h \) is infinitesimal) must be the value of \( \frac{d\phi}{dy} \) for the point \((0, y)\), if \( \phi \) denote the velocity-potential at any point \((x, y)\) of the water. Now because

\[ \frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} = 0, \]
145. Kelvin-Helmholtz instability of stratified shear flow. A long rectangular tube, initially horizontal, is filled with water above colored brine. The fluids are allowed to diffuse for about an hour, and the tube then quickly tilted six degrees, setting the fluids into motion. The brine accelerates uniformly down the slope, while the water above similarly accelerates up the slope. Sinusoidal instability of the interface occurs after a few seconds, and has here grown nonlinearly into regular spiral rolls. Thorpe 1971
117. Instability of a round jet. This shadowgraph shows a 3/4-inch jet of carbon dioxide issuing into air at a speed of 127 ft/s. It is laminar as it leaves the nozzle at a Reynolds number of approximately 30,000. One diameter downstream it shows instability, formation of vortex rings, and transition to turbulence. Photograph by Fred Landis and Ascher H. Shapiro
Large-scale structure in a turbulent mixing layer. Nitrogen above flowing at 1000 cm/s mixes with a helium-argon mixture below at the same density flowing at 380 cm/s under a pressure of 4 atmospheres. Spark shadow photography shows simultaneous edge and plan views, demonstrating the spanwise organization of the large eddies. The streamwise streaks in the plan view (of which half the span is shown) correspond to a system of secondary vortex pairs oriented in the streamwise direction. Their spacing at the downstream side of the layer is larger than near the beginning. Photograph by J. H. Konrad, Ph.D. thesis, Calif. Inst. of Tech., 1976.
Periodic/linear vs localized/nonlinear
The classical picture (Kelvin)

The picture inspired by Helmholtz

FIG. 1 (color online). Kelvin-Helmholtz instability excited by two types of initial perturbation, shown by the deformation of the interface. We = 1000, Re = 100. The two fluids have the same density \((r = 1)\). For the wavelike initial condition, we observe the familiar roll-up of the interface in vortices whose size is fixed by the initially imposed wavelength. For the localized initial perturbation, we observe the creation of a system of two vortices growing in size without alteration in shape.
Movie of the excited jet
**Dimensional parameters**

**Geometry:** shear layer thickness $L$

The initial length scale

The self-similar length scale

Gas:
- density, viscosity, velocity

Liquid:
- density, viscosity, velocity

Viscosity ratio: $n$
Density ratio: $r$
Velocity difference: $\Delta U$

$$\omega = \frac{U}{\delta} f\left(\frac{x}{Ut}, \frac{y}{Ut'}, \frac{\rho_{\text{gas}}}{\rho_{\text{liq}}}, \frac{\delta}{Ut'}, \frac{L_0}{Ut}\right)$$
Representation of the vorticity
Jimenez’ search for a self-similar rollup of the vortex sheet.

**Fig. 1.** The normalized coordinates for the vortex sheet equation.

**Fig. 2.** Numerical evolution of the discrete vortex sheet after a finite initial disturbance. No extra vorticity at the origin.

**Fig. 3.** The growth in time of the vortices in the numerical experiments. The straight line is the observed growth of physical layers.
Pullin’s self-similar vortex sheet roll-up family
Comparison: Gerris simulation and Pullin’s self-similar solution.
A solution with two vortices
A self-similar model

Jimenez:

Fig. 5. A simplified version of the rolled-up sheet.
Bassin of attraction

For several initial position of the red vortex
Kaden’s self-similar vortex sheet roll-up

The sheet is semi-infinite and its strength fades away from the tip.
Atomisation
Waves on water jets

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The structure of atomization

Figure 2. Jet emerging from 0.25 in. diameter nozzle into stagnant air.

Jet velocity = 83 ft/s.
Analysis by linear theory:
- inviscid Kelvin-Helmholtz
- viscous H mode (Hooper&Boyd /Hinch)
- even more sophisticated?

Figure 3
Snapshots of an 8-mm-diameter, slow (0.6 m/s) water jet destabilized by a coaxial fast air stream. Development of the axisymmetric shear instability, digitations at the wave crests, and ligament formation for air velocities increasing from 20 to 60 m/s are shown (Marmottant & Villermaux 2004).
Fast gas

Splitter plate

Slow liquid

Simulation of the Navier-Stokes equations using the Gerris Flow Solver (open source)
FIG. 4 (color online). Evolution of the wave represented as a spatiotemporal diagram for three representative density ratios. The characteristic cone delimited from the virtual origin by the liquid and gas speed is represented as a frame of reference. The speed of advancement of the wave is compared to the Dimotakis speed, characteristic of propagation in this two-phase system. The origin of the self-similar shape \((x_0, t_0)\) is visualized as the tip of the characteristic cone. We observe that it differs from \((0,0)\). This effect is due to the transient from the initial forcing.
Gravity and
the oceans
La notion de « vague spontanée de tempête »

BEAUFORT FORCE 11
WIND SPEED: 56-63 KNOTS

SEA: WAVE HEIGHT 11.5-16M (37-52FT), EXCEPTIONALLY HIGH WAVES, SMALL-MEDIUM SIZED SHIPS MAY BE LOST TO VIEW BEHIND THE WAVES. SEA COMPLETELY COVERED WITH LONG WHITE PATCHES OF FOAM LYING ALONG WIND DIRECTION. EVERYWHERE, THE EDGES OF WAVE CRESTS ARE BLOWN INTO FROTH.

BEAUFORT FORCE 12
WIND SPEED: 64 KNOTS

SEA: SEA COMPLETELY WHITE WITH DRIVING SPRAY, VISIBILITY VERY SERIOUSLY AFFECTED. THE AIR IS FILLED WITH FOAM AND SPRAY
Theoretical model of the wave with gravity

- **Bernoulli equation:**
  - gas: \( P^+ - P^- \propto \rho_{gas} U^2 \)
  - liquid: \( P^+ - P^- \propto \rho_{liq} V^2 + \rho_{liq} gL \)

- **Conservation of Mass:**
  \[ \partial_t A \propto LV \]

Differential equation for the wave size

\[ \partial_t L \propto V \quad \Rightarrow \quad \begin{cases} \partial_t L = \sqrt{C \cdot rU^2 - B \cdot gL(t)} \\ L(0) = L_0 \\ \lim_{t \to \infty} L(t) = L_{grav} \end{cases} \]

- \( B \) and \( C \) are two constant to determine.
- \( C \cdot rU^2 \) is the term that linearly growth in time.
- \( B \cdot gL(t) \) is the new term that stops the linear growth of the wave size.
The diagram illustrates the growth of a wave over time for different values of gravity, denoted as $g$. The growth is categorized into two types:

- **Algebraic growth, $g=0$**
- **Increase gravity**

Each curve represents a different gravity value, with $g=0$ being the algebraic growth and $g=0.001$, $g=0.003$, $g=0.005$, $g=0.007$, $g=0.01$, and $g=0.02$ indicating increasing gravity. The $y$-axis represents the size of the wave, and the $x$-axis represents time.