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Rapid Note

Cluster formation, pressure and density measurements in a granular medium fluidized by vibrations

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Abstract. We report experimental results on the behavior of an ensemble of inelastically colliding particles, excited by a vibrated piston in a vertical cylinder. When the particle number is increased, we observe a transition from a regime where the particles have erratic motions ("granular gas") to a collective behavior where all the particles bounce like a nearly solid body. In the gas-like regime, we measure the density of particles as a function of the altitude and the pressure as a function of the number N of particles. The atmosphere is found to be exponential far enough from the piston, and the "granular temperature", T, dependence on the piston velocity, V, is of the form $T \propto V^{\theta}$, where θ is a decreasing function of N. This may explain previous conflicting numerical results.

PACS. 45.70.-n Granular systems – 83.70.Fn Granular solids – 05.20.Dd Kinetic theory – 83.10.Pp Particle dynamics

Granular matter is an interdisciplinary subject, involving soil mechanics (rheology), powder technology [1,2], geophysics (dunes [3], ice floes [4]), astrophysics (planetary rings [5]) and statistical physics of dissipative media [6,7]. Recently, considerable attention has been devoted to the role of the inelasticity of the collisions in vibrated granular media, the so-called driven "granular gas" for which the stationary state results from the balance between dissipation by inelastic collisions and power input by external vibrations. While over the years many attempts based on kinetic theory [8–11] have been made to describe such dissipative granular gases, no agreement has been found so far both with experiments [12,13] and numerical simulations [13–15], for the dependence of the "granular temperature" with the parameters of vibration [16–18]. The aim of this study was to guess possible gas-like state equations for such dissipative granular gas and to observe new kinetic behaviors which trace back to the inelasticity of collisions.

We report an experimental study of a "gas" of inelastically colliding particles, excited by vertical vibrations. When the vibration is strong enough and the number of particles is low enough, the particles display ballistic motion between successive collisions like molecules in a gas (see Fig. 1a). At constant external driving, we show that the pressure passes through a maximum for a critical number of particles before decreasing for large N. We also measure density profiles and extract granular temperature from them. When the density of the medium is increased, the gas-like state is no longer stable but displays the formation of a dense cluster bouncing like a nearly solid body (see Fig. 1b).

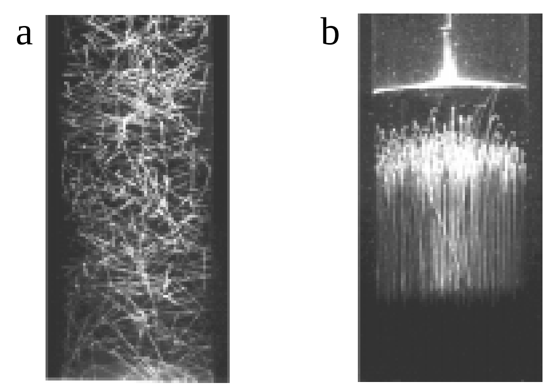

Fig. 1. Transition from a dissipative granular gas to a dense cluster: (a) $N = 480$; (b) $N = 1920$, respectively corresponding to roughly 1 and 4 particle layers at rest. The parameters of vibration are $f = 20$ Hz and $A = 40$ mm. The driving piston is at the bottom (not visible), the inner diameter of the tube being 52 mm.

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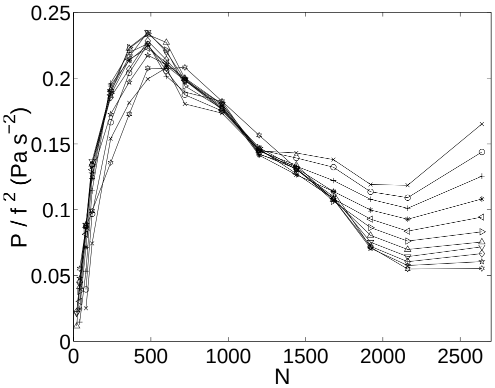

Fig. 2. Mean pressure P rescaled by f^2 as a function of N. From the upper (×-marks) to the lower (hexagrams) curve, vibration frequency f varies from 10 to 20 Hz with a 1 Hz step. For all these experiments, $h - h_0 = 5$ mm and $A = 25$ mm. One single layer of particles at rest corresponds to $N = 600$. Lines join the data points.

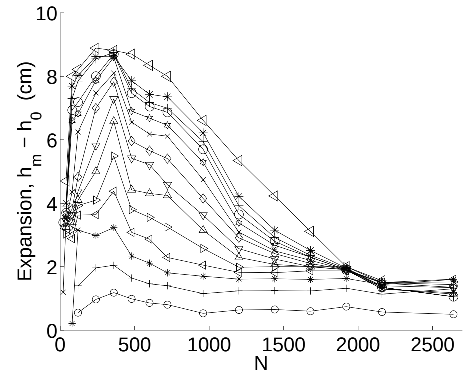

Fig. 3. Maximal bed expansion, $h_m - h_0$, as a function of N, for various frequencies f of vibration. From the lower (∘-marks) to the upper (◁-marks) curve, f varies from 7 to 20 Hz with a 1 Hz step and $A = 25$ mm. One single layer of particles at rest corresponds to $N = 600$. Lines join the data points.

The experiment consists of a transparent cylindrical tube, 60 mm in inner diameter, filled from 20 to 2640 stainless steel spheres, 2 mm in diameter, roughly corresponding to no more than 5 particle layers at rest. An electrical motor, with eccentric transformer from rotational to translational motion, drives the particles sinusoidally with a 25 or 40 mm amplitude, A, in the frequency range from 6 to 20 Hz. A lid in the upper part of the cylinder, is either fixed at a given height, h (constant-volume experiment) or is stabilized at a given height h_m due to the bead collisions (constant-pressure experiment). Heights h and h_m are defined from the lower piston at full stroke.

Time averaged pressure measurements have been done as follows. Initially, a counterweight of mass $m = 46$ g balances the lid mass. The piston drives stainless steel spheres in erratic motions in all directions (see Fig. 1a). Particles are hitting the lid all the time, so that to keep it at a given height h we have to hold the lid down by a given force, Mg, where M is the mass of a weight we place on the lid and g the acceleration of gravity. At a fixed h, *i.e.* at a constant-volume, Figure 2 shows the time averaged pressure Mg/S exerted on the lid as a function of the number N of beads in the container, for different frequencies of vibration, S being the area of the tube cross-section. At constant external driving, *i.e.* at fixed f and A, the pressure passes through a maximum for a critical value of N roughly corresponding to 0.8 particle layers at rest. This critical number is independent on the vibration frequency. A further increase of the number of particles leads to a decrease in the mean pressure underlying that more and more energy is dissipated by inelastic collisions. Note that gravity has a small effect in these measurements that are performed for $V^2 \gg gh$, where $V = 2\pi fA$ is the maximum velocity of the piston. For N such that one has less than one particle layer at rest, most particles are in vertical ballistic motion between the piston and the lid. Thus, the mean pressure increases roughly proportionally to N. When N is increased such that one has more than one particle layer at rest, interparticle collisions become more frequent. The energy dissipation is increased and thus the pressure decreases. In the low density regime (less than two particles layers at rest) the mean pressure scales like the square of the piston velocity, V^2.

We now consider the bed expansion under the influence of collisions on a circular wire mesh lid placed on top of the beads leaving a clearance of about 0.5 mm between the edge of the lid and the tube one. Due to the bead collisions, the lid is stabilized at a given height h_m from the piston at full stroke. Although the lid mass is roughly 50 times smaller than the total mass of beads, the lid proves to be quite stable and remains horizontal. The expansion, $h_m - h_0$, of the bed is displayed in Figure 3 as a function of N for different vibration frequencies. h_0 is the bed height at rest. At fixed f, the expansion passes through a maximum for a critical value of N roughly corresponding to 0.6 particle layers at rest. This critical number is independent on the vibration frequency. When N is further increased, the expansion decreases showing, as for pressure measurements, an increase in dissipated energy by inelastic collisions. Note that the height h_m of the granular gas is much larger than for pressure measurements of Figure 2. Consequently, gravity is obviously important. Moreover, for a given N, the number of interparticle collisions is larger than for the pressure measurements. One cannot consider that most particles are in ballistic motion between the piston and the lid, and thus the V^2 scaling is no longer observed.

As already found experimentally in 1-D [13] and 3-D [19,20] and numerically [13,21], we observe that, at N fixed, the granular medium exhibits (not shown here) a sudden expansion at a critical frequency corresponding to a bifurcation similar to that exhibited by a single ball bouncing on a vibrating plate [13,19]. Moreover, we find that this critical frequency depends on the number of layers, n. When n increases above 0.4, a transition from the 1-D-like behavior to a 3-D one is observed: the expansion at the critical frequency becomes less abrupt and tends to increase regularly with f.

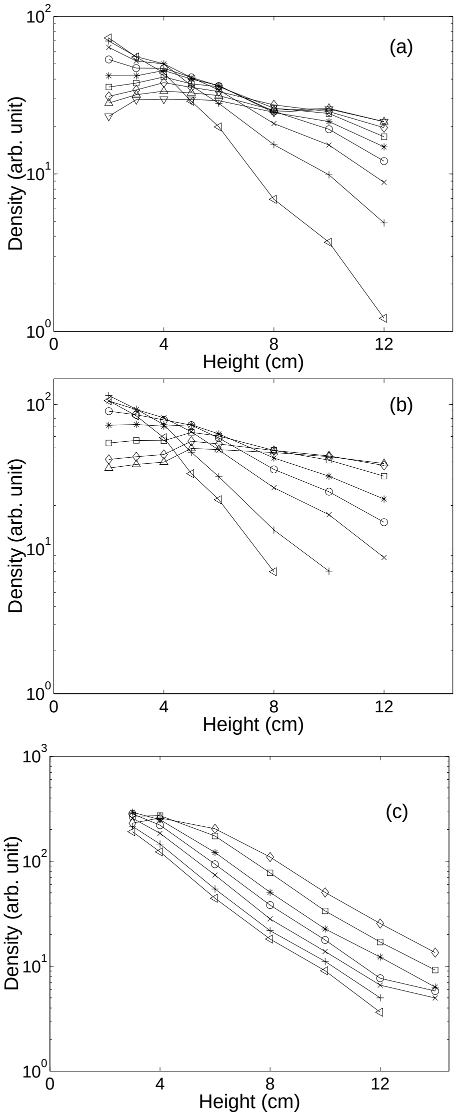

Fig. 4. Mean density as a function of the height, for various frequencies f of vibration and 3 numbers of particles: (a) $N = 480$; $f = 5$ (◁), 6 (+), 7 (×), 8 (○), 9 (∗), 10 (□), 11 (◇), 12 (△) and 13 Hz (▽); (b) $N = 720$; $f = 5$ (◁), 5.6 (+), 7.1 (×), 8.6 (○), 10.1 (∗), 12.2 (□), 14 (◇) and 15.2 Hz (△); (c) $N = 1440$; $f = 7$ (◁), 8 (+), 10 (×), 11.4 (○), 12.8 (∗), 15 (□) and 17 Hz (◇). For all these experiments, $A = 40$ mm. One single layer of particles at rest corresponds to $N = 480$. Lines join the data points.

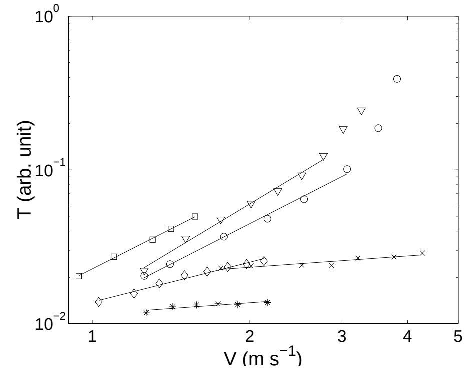

Fig. 5. Log-log plot of granular temperature *versus* V for various number of layers: (▽) 0.8, (□) 1, (○) 1.2, (◇) 2, (×) 2.4 and (∗) 3. Experiments (□), (◇) and (∗) (resp. (▽), (○) and (×)) are performed for $A = 25$ mm (resp. $A = 40$ mm). Power law fits of the form V^θ are displayed in full lines.

Time averaged density measurements at a given height z are performed by means of two closely coupled coils, $\delta z = 5$ mm in height, and 64 mm in inner diameter, the cylindrical tube now being 52 mm (resp. 62 mm) in inner (resp. outer) diameter. An 1.5 kHz a.c. voltage is applied to the primary coil, the turns ratio of the transformer being roughly equal to 2. Steel spheres moving across the permanent magnetic field of the primary coil, generate an inductive voltage variation across the secondary one. This root mean square a.c. voltage Δe is a function of the mean inductance and mutual variations which are proportional to the mean number of particles in the volume delimited by the sensor at altitude z from the piston surface at full stroke. We have calibrated the sensor with steel spheres at rest and we have checked that $\Delta e \propto N$. We have also found that the effect of spheres outside of the sensor volume decays exponentially with the distance to the sensor, with a 10 ± 1 mm decay length independent of N, for our range of N. Particles density as a function of altitude is shown in Figures 4a–4c for 3 different total numbers of particles and for various frequencies. For each N, the profile density in log-linear axes displays a decay (at low f), a plateau (at intermediate f) or a dip (at high f) near the piston and an exponential decay in the tail at high altitude, whatever f.

As usual gas, the atmosphere is found to be exponential far enough from the piston, but on very different length scales, *i.e.* few cm (resp. km) for our experiment (resp. for air). Such a dense upper region supported on a fluidized low-density region near the piston is also reported numerically [21] and theoretically [6]. Although the dip in the density profiles at the bottom was already observed in a 2-D granular gas experiment, non-negligible coherent friction force acting on all the particles did not allow determination of the granular temperature dependence on the piston velocity V from exponential Boltzmann distributions fitted to tails of density profiles [12]. We can fit an exponential curve to the tail of the profile density. From the decay rate α in the fitted exponential and using kinetic theory [12], we can extract the dependence for the granular temperature T on the piston velocity: plotting as in Figure 5, in log-log axes, $-1/\alpha$, which is proportional to T, *versus* V then shows power law fits of the form $T \propto V^\theta$ where θ is n dependent. Note that this power law, being observed only on a small range of velocities, one cannot rule out another functional behavior. In particular, the faster increase of T at high velocity is not significant because of imprecision on the exponential decay of the density (see Fig. 4). Values of θ are extracted from the power law fits in Figure 5 and are displayed in Figure 6 as a function of the number of layers. Figure 6 shows that the exponent θ decreases when the number of layers increases.

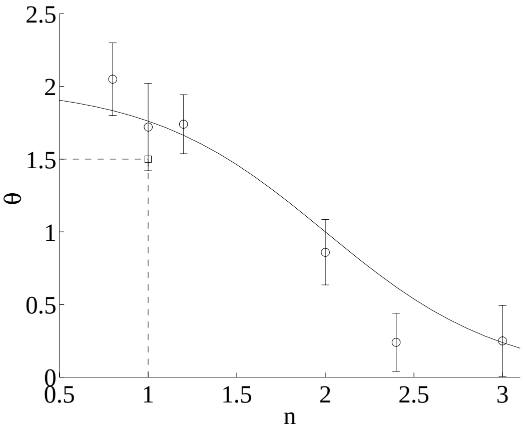

Fig. 6. Evolution of the θ exponent in $T \propto V^\theta$ as a function of the number of layers, n. The $V^{3/2}$ scaling (□-mark) for $n = 1$ is from reference [24] performed in low gravity. See the text for details.

The dependence of the granular temperature on the piston velocity has been addressed in various papers, using kinetic theory [12,18] and hydrodynamics models [16]. A V^2 scaling law has been predicted in low-density regime and a $V^{4/3}$ in high-density one [22], whereas a *transition* from V^2 to $V^{3/2}$ is reported numerically when the granular medium is not enough fluidized [17]. When n increases until one layer, others numerical simulations have reported a *continuous* dependence from $\theta = 2$ to $\theta = 3/2$ [15, 23], whereas no dependence has been observed in a 2-D experiment from height of center of mass measurements (see Fig. 14 of [12] or Ref. [23]). For $0 < n \leq 1$, our results are thus in agreement with the ones in reference [15,23] and extend them for $n > 1$, since the exponent almost vanishes for large n (see Fig. 6). Note that our previous $V^{3/2}$ scaling found in low-gravity for $n = 1$ [24] is thus coherent with those results (see the □-mark in Fig. 6). For small n, θ slowly decreases from its value $\theta = 2$ predicted by kinetic theory, as n is increased. For large n, the granular temperature is almost independent of V, thus $\theta \simeq 0$. One of the simplest empirical law fitting this behavior is $\theta = \tanh(n - n_c)$ with $n_c = 2$ (full line in Fig. 6).

The particle aggregation phenomenon displayed in Figure 1b seems similar to the various clustering phenomena observed numerically [25]. A cluster formation has been also observed in a 2-D experiment, with a horizontally shaken layer of particles, but the coherent friction force acting on all the particles was far from being negligible [26]. We have shown in this paper that a 3-D granular gas excited by a vibrating piston displays a transition from a kinetic regime with an exponential atmosphere far enough from the piston, to a cluster that bounces as a nearly solid body, when the particle density is increased.

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