State of the art in wind turbine aerodynamics and aeroelasticity

M.O.L. Hansen\textsuperscript{a,*}, J.N. Sørensen\textsuperscript{a}, S. Voutsinas\textsuperscript{b}, N. Sørensen\textsuperscript{c,d}, H.Aa. Madsen\textsuperscript{c}

\textsuperscript{a}Department of Mechanical Engineering, Technical University of Denmark, Fluids Section, Nils Koppels Alle, Building 403, DK-2800 Lyngby, Denmark
\textsuperscript{b}Department of Mechanical Engineering, National Technical University of Athens, Fluids Section, 15780 Zografou, Greece
\textsuperscript{c}Wind Energy Department, Risø National Laboratory, Building VEA-762, P.O. Box 49, Frederiksbergvej 399, DK-4000 Roskilde, Denmark
\textsuperscript{d}Department of Civil Engineering, Aalborg University, Sohngaardsholmvej 57, DK 9000 Aalborg, Denmark

Available online 29 December 2006

Abstract

A comprehensive review of wind turbine aeroelasticity is given. The aerodynamic part starts with the simple aerodynamic Blade Element Momentum Method and ends with giving a review of the work done applying CFD on wind turbine rotors. In between is explained some methods of intermediate complexity such as vortex and panel methods. Also the different approaches to structural modelling of wind turbines are addressed. Finally, the coupling between the aerodynamic and structural modelling is shown in terms of possible instabilities and some examples.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Aeroelasticity; Wind turbines

Contents

1. Introduction ................................................................. 286
2. Predicting aerodynamic loads on a wind turbine. .................. 287
   2.1. Blade Element Momentum Method .................................. 287
       2.1.1. Dynamic wake/inflow ........................................... 288
       2.1.2. Yaw/tilt model .................................................. 289
       2.1.3. Dynamic stall .................................................. 289
       2.1.4. Airfoil data .................................................... 290
       2.1.5. Wind simulation ................................................. 290
   2.2. Lifting line, panel and vortex models .......................... 291
       2.2.1. Vortex methods ............................................... 291
       2.2.2. Panel methods ............................................... 292
   2.3. Generalized actuator disc models .................................. 295
   2.4. Navier–Stokes solvers .............................................. 297
       2.4.1. Introduction to computational rotor aerodynamics ........ 297
       2.4.2. Approaches ................................................... 298
       2.4.3. Turbulence and transition ................................... 299

*Corresponding author. Tel.: +45 45254316.
E-mail address: molh@mek.dtu.dk (M.O.L. Hansen).

0376-0421/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
1. Introduction

The size of commercial wind turbines has increased dramatically in the last 25 years from approximately a rated power of 50 kW and a rotor diameter of 10–15 m up to today’s commercially available 5 MW machines with a rotor diameter of more than 120 m. This development has forced the design tools to change from simple static calculations assuming a constant wind to dynamic simulation software that from the unsteady aerodynamic loads models the aeroelastic response of the entire wind turbine construction, including tower, drive train, rotor and control system. The Danish standard DS 472 [1] allows simplified load calculations if the rotor diameter is less than 25 m and some other criteria are fulfilled. A rotor diameter of 25 m corresponds approximately to a rated power of 200–250 kW, which is less than almost any modern commercial wind turbine today. Instead, modern wind turbines are designed to fulfill the requirements of the more comprehensive IEC 61 400-1 [2] standard. At some time during the development of larger and larger commercial wind turbines the need for aeroelastic tools thus became necessary. Aeroelastic tools were mainly developed at the universities and research laboratories in parallel with the evolution of commercial wind turbines. At the same time governments and utility companies erected large non-commercial prototypes for research purposes, as the Nibe [3] and Tjaereborg machines [4]. Measurement campaigns were undertaken on these machines and the results used to tune and validate the aeroelastic programmes, in order to develop advanced software for the rapidly growing industry. Even today measurements from the Tjaereborg machine is used as a benchmark when developing new aeroelastic codes, see e.g. [5]. In [5] is also compiled a long list of available software that at different levels of complexity can model the aeroelastic response of a wind turbine construction. All the aeroelastic models need as input a time history of the wind seen by the rotor, which as a minimum must contain some physical correct properties such as realistic power spectra and spatial coherence. Apart from the wind input aeroelastic codes contain
an aerodynamic part to determine the wind loads and a structural part to describe the dynamic response of the wind turbine construction. For the aerodynamic part most codes use the Blade Element Momentum Method (BEM) as described by Glauert [6], since this method is very fast and, provided that reliable airfoil data exist, yields accurate results. Therefore, this method, with all the necessary engineering adds on, is thoroughly described later in this article. However, more advanced numerical models based on the Euler and Navier–Stokes (NS) equations are becoming so fast that they now begin to replace the BEM method in some situations, e.g. when analysing yaw or interaction between wind turbines in parks. These models contain more physics and less empirical input than the BEM method and are extensively described in this paper. The discretization of the wind turbine structure is presently where the various available codes differ most. Roughly, there exist three ways to model the structural dynamics of a wind turbine. One is a full Finite Element Method (FEM) discretization and another is a multi-body formulation, where different rigid parts are connected through springs and hinges. Finally, the description of blade and tower deflections can be made as a linear combination of some physical realistic modes; typically the lowest eigenmodes. The last method greatly reduces the computational time per time step, as compared with a full FEM discretization. All the various ways of discretizing the wind turbine structure will be treated in details later in the paper. The very detailed description of the aerodynamic and structural models is where this paper differs mostly from other review articles concerning wind turbine aeroelasticity such as e.g. [7–9].

2. Predicting aerodynamic loads on a wind turbine

Methods of various levels of complexity to calculate the aerodynamic loads on a wind turbine rotor are given, starting with the popular BEM, and ending with the solution of the NS equations.

2.1. Blade Element Momentum Method

BEM is the most common tool for calculating the aerodynamic loads on wind turbine rotors since it is computationally cheap and thus very fast. Further, it provides very satisfactory results provided that good airfoil data are available for the lift and drag coefficients as a function of the angle of attack, and if possible, the Reynolds number. The method was introduced by Glauert [6] as a combination of one-dimensional (1D) momentum theory and blade element considerations to determine the loads locally along the blade span. The method assumes that all sections along the rotor are independent and can be treated separately; typically in the order of 10–20 radial sections are calculated. At a given radial section a difference in the wind speed is generated from far upstream to deep in the wake. The resulting momentum loss is due to the axial loads produced locally by the flow passing the blades, creating a pressure drop over the blade section. The local angle of attack at a given radial section on a blade can be constructed, provided that the induced velocity generated by the action of the loads is known, see Fig. 1. \( V_0 \) is the undisturbed wind velocity, \( W \), the induced velocity, \( V_{rot} = \omega \cdot r \) the rotational speed of the blade section, \( V_{blade} \) the velocity of the blade section apart from the blade rotation and \( \beta \) is the local angle of the blade section to the rotor plane.

Combining the global momentum loss with the loads generated locally at the blade section yields formulas for the induced velocity as

\[
W_z = \frac{-BL \cos \phi}{4 \rho \pi r F \left| V_0 + f \cdot \mathbf{n} \cdot \mathbf{W} \right|},
\]

(2.1.1)

\[
W_y = \frac{-BL \sin \phi}{4 \rho \pi r F \left| V_0 + f \cdot \mathbf{n} \cdot \mathbf{W} \right|},
\]

(2.1.2)

\( B \) is the number of blades, \( L \) the lift computed from the lift coefficient, \( \phi \) is the flow angle, \( \rho \) the density of air, \( r \) the radial position considered, \( V_0 \) the wind velocity, \( W \) the induced velocity and \( \mathbf{n} \) the normal vector to the rotor plane. \( F \) is Prandtl’s tip loss correction that corrects the equations to be valid for a finite number of blades, see [6, 10]. If there is no yaw misalignment, that is, the normal vector to the rotor plane, \( \mathbf{n} \), is parallel to the wind vector, then
Eq. (2.1.1) reduces to the well-known expression
\[ C_T = 4aF(1 - f_g \cdot a), \] (2.1.3)
where by definition for an annual element of infinitesimal thickness, \( dr \), and area, \( dA = 2\pi r \, dr \),
\[ C_T = \frac{dT}{1/2\rho V_0^2 \, dA}. \] (2.1.4)
The axial interference factor is defined as
\[ a = \frac{W_z}{V_0} \] (2.1.5)
and \( f_g \), usually referred to as the Glauert correction, is an empirical relationship between \( C_T \) and \( a \), in the turbulent wake state. It may assume the form
\[ f_g = \begin{cases} 
1 & \text{for } a \leq 0.3, \\
\frac{4}{3}(5 - 3a) & \text{for } a > 0.3.
\end{cases} \] (2.1.6)
Eqs. (2.1.1) and (2.1.2) are also known to be valid for an extreme yaw misalignment of 90°, that is, the incoming wind is parallel to the rotor plane as a helicopter in forward flight. Without any proof Glauert therefore assumed that Eqs. (2.1.1) and (2.1.2) are valid for any yaw angle.

An aeroelastic code is running in the time domain and for every time step the aerodynamic loads must be calculated at all the chosen radial stations along the blades as input to the structural model. For a given time the local angle of attack is determined on every point on the blades, as indicated in Fig. 1. The lift and drag coefficients can now be found from table look-up, and the lift can be determined. The induced velocities can now be updated using Eqs. (2.1.1) and (2.1.2) simply assuming old values for the induced velocities on the right-hand sides (RHS). Updating the RHS of Eqs. (2.1.1) and (2.1.2) could continue until the equations are solved with all values at the same time step. However, this is not necessary as this update takes place in the next time step, i.e. time acts as iteration. More important, the values of the induced velocities change very slowly in time due to the phenomena of dynamic inflow or dynamic wake.

### 2.1.1. Dynamic wake/inflow

The induced velocities calculated using Eqs. (2.1.1) and (2.1.2) are quasi-steady, in the sense that they give the correct values only when the wake is in equilibrium with the aerodynamic loads. If the loads are changed in time there is a time delay proportional to the rotor diameter divided by the wind speed before a new equilibrium is achieved. To take into account this time delay, a dynamic inflow model must be applied. In two EU-sponsored projects ([11,12]) different engineering models were tested against measurements. One of these models, proposed by S. Øye, is a filter for the induced velocities consisting of two first-order differential equations
\[ W_{\text{int}} + \tau_1 \frac{dW_{\text{int}}}{dt} = W_{qs} + k \cdot \tau_1 \frac{dW_{qs}}{dt}, \] (2.1.7)
\[ W + \tau_2 \frac{dW}{dt} = W_{\text{int}}, \] (2.1.8)
\( W_{qs} \) is the quasi-static value found by Eqs. (2.1.1) and (2.1.2), \( W_{\text{int}} \) an intermediate value and \( W \) the final filtered value to be used as the induced velocity. The two time constants are calibrated using a simple vortex method as
\[ \tau_1 = \frac{1.1}{(1 - 1.3a)} \cdot \frac{R}{V_0} \] (2.1.9)
and
\[ \tau_2 = \left( 0.39 - 0.26 \left( \frac{R}{R} \right)^2 \right) \tau_1, \] (2.1.10)
where \( R \) is rotor radius.

In Fig. 2 is shown for the Tjaereborg machine the computed and measured response on the rotorshaft torque for a sudden change of the pitch angle. At \( t = 2 \) s the pitch is increased from 0° to 3.7°, decreasing the local angles of attack. First the rotorshaft torque drops from 260 to 150 kNm and not until approximately 10 s later the induced velocities and thus the rotorshaft torque have settled.
at a new equilibrium. At \( t = 32 \text{s} \) the pitch is changed back to 0° and a similar overshoot in rotorshaft torque is observed. The decay of the spikes seen in Fig. 2 can only be computed with a dynamic inflow model, and such a model is therefore of utmost importance for a pitch-regulated wind turbine.

2.1.2. Yaw/tilt model

Another engineering model for the induced velocities concerns yaw or tilt. When the rotor disc is not perfectly aligned with the incoming wind there is an angle different from zero between the rotor normal vector and the incoming wind, see Fig. 3. A yaw/tilt model redistributes the induced velocities so that the induced velocities are higher when a blade is positioned deep in the wake than when it is pointing more upstream. An example of such a model, taken from helicopter literature [13] is given below. Here, the input is the induced velocity, \( W_0 \), calculated using Eqs. (2.1.1), (2.1.2), (2.1.7) and (2.1.8). The output is a redistributed value, finally used when estimating the local angle of attack, \( W \),

\[
W = W_0 \left( 1 + \frac{r}{R} \tan \frac{\chi}{2} \cos(\theta_b - \theta_0) \right), \tag{2.1.11}
\]

\( \theta_b \) is the actual position of a blade, \( \theta_0 \), is the position where the blade is furthest downstream and \( \chi \) is the wake skew angle, see Fig. 3. In some BEM implementations, \( W_0 \), is the average value of all blades at the same radial position, \( r \), and in other codes it is the local value. This difference in implementation may cause a small difference from code to code. Further, there exist different modifications of Eq. (2.1.11) from different codes, see [12]. A yaw/tilt model increases the induced velocities on the downstream part of the rotor and decreases similarly the induced velocity on the upstream part of the rotor disc. This introduces a yaw moment that tries to align the rotor with the incoming wind, hence tending to reduce yaw misalignment. For a free yawing turbine such a model is, therefore, of utmost importance when estimating the yaw stability of the machine.

2.1.3. Dynamic stall

The wind seen locally on a point on the blade changes constantly due to wind shear, yaw/tilt misalignment, tower passage and atmospheric turbulence. This has a direct impact on the angle of attack that changes dynamically during the revolution. The effect of changing the blades angle of attack will not appear instantaneously but will take place with a time delay proportional to the chord divided with the relative velocity seen at the blade section. The response on the aerodynamic load depends on whether the boundary layer is attached or partly separated. In the case of attached flow the time delay can be estimated using Theodorsen theory for unsteady lift and aerodynamic moment [14]. For trailing edge stall, i.e. when separation starts at the trailing edge and gradually increases upstream at increasing angles of attack, so-called dynamic stall can be modelled through a separation function, \( f_s \), as described in [15], see later. The Beddoes–Leishman model [16] further takes into account attached flow, leading edge separation and compressibility effects, and also corrects the drag and moment coefficients. For wind turbines, trailing edge separation is assumed to represent the most important phenomenon regarding dynamic airfoil data, but also effects in the linear region may be important, see [17]. It is shown in [15] that if a dynamic stall model is not used one might compute flapwise vibrations, especially for stall regulated wind turbines, which are non-existing on the real machine. For stability reasons it is thus highly recommended to at least include a dynamic stall model for the lift. For trailing edge stall the degree of stall is described through \( f_s \), as

\[
C_l(\alpha) = f_s C_{l,\text{inv}}(\alpha) + (1 - f_s) C_{l,\text{fs}}(\alpha), \tag{2.1.12}
\]

where \( C_{l,\text{inv}} \) denotes the lift coefficient for inviscid flow without any separation and \( C_{l,\text{fs}} \) is the lift coefficient for fully separated flow, e.g. on a flat plate with a sharp leading edge. \( C_{l,\text{inv}} \) is normally an extrapolation of the static airfoil data in the linear

\[\text{Fig. 3. Wind turbine rotor not aligned with the incoming wind.}\]

The angle between the velocity in the wake (the sum of the incoming wind and the induced velocity normal to the rotor plane) is denoted the wake skew angle, \( \chi \).
region, and in [17] a way of estimating $C_{l,lib}$ and $f_s^d$ is shown. $f_s^d$ is the value of $f_s$ that reproduces the static airfoil data when applied in Eq. (2.1.12). The assumption is that, $f_s$, always will try to get back to the static value as

$$\frac{df_s}{dt} = \frac{f_s^{st} - f_s}{\tau},$$  

(2.1.13)

that can be integrated analytically to give

$$f_s(t + \Delta t) = f_s^{st} + (f_s(t) - f_s^{st}) \exp(-\Delta t/\tau),$$  

(2.1.14)

$\tau$ is a time constant approximately equal to $Ac/V_{rel}$, where $c$ denotes the local chord, and $V_{rel}$ is the relative velocity seen by the blade section. $A$ is a constant that typically takes a value about 4. Applying a dynamic stall model the airfoil data is always chasing the static value at a given angle of attack that is also changing in time. If e.g. the angle of attack is suddenly increased from below to above stall the unsteady airfoil data contains for a short time some of the inviscid/unstalled value, $C_{l,inv}$, and an overshoot relative to the static data is seen. It can thus been seen as a model of the time constant for the viscous boundary layer to develop from one state to another.

### 2.1.4. Airfoil data

The BEM as described above, including all engineering corrections, is used in most aeroelastic codes to compute the unsteady aerodynamic loads on wind turbine rotors. The method is often quite successful, but depends on reliable airfoil data for the different blade sections. Three-dimensional (3D) effects from the tip vortices are taken into account when applying Prandtl's tip loss correction and after this correction the local flow around a given blade section is assumed to be two-dimensional, i.e. 2D airfoil data from wind tunnel measurements are used. However, such measurements are often limited to the maximum lift coefficient, $C_{l,\text{max}}$ for airfoils that usually are operated at unstalled flow conditions. Further, at higher values it is difficult to measure the forces because of the unsteady and 3D nature of stall. In contrast to airplane wings, a wind turbine blade often operates in deep stall, especially for stall regulation. For the inner part of the blades even data for low angles of attack might be difficult to find in literature since for structural reasons the airfoils used are much thicker than those used on airplanes. Further, because of rotation the boundary layer is subjected to Coriolis- and centrifugal forces, which alter the 2D airfoil characteristics. This is especially pronounced in stall. It is thus often necessary to extrapolate existing airfoil data into deep stall and to include the effect of rotation. Methods have been developed that from a CFD calculation of the flow past a full wind turbine rotor can extract 3D airfoil data [18], which then later can be applied in aeroelastic calculations using the much faster BEM method. In this method, the induced velocity at the rotor plane is estimated from the azimuthally averaged velocity in very thin annular elements up- and downstream of the rotorplane. In [19,20], two engineering methods to correct 2D airfoil data for 3D rotational effects are given as

$$C_{x,3D} = C_{x,2D} + a(c/r)^b \times \cos^n \beta \times \Delta C_x, \quad x = l, d, m,$$

(2.1.15)

$$\Delta C_l = C_{l,\text{inv}} - C_{l,2D},$$

$$\Delta C_d = C_{d,2D} - C_{d,\text{2D-min}}$$

$$\Delta C_m = C_{m,2D} - C_{m,\text{inv}},$$

c is the chord, $r$ the radial distance to rotational axis and $\beta$ the twist.

In [19] only the lift is corrected, i.e. $x = l$, and the constants are $a = 3$, $n = 0$ and $h = 2$, whereas in [20] $a = 2.2$, $n = 4$ and $h = 1$. In [21] another method based on correcting the pressure distribution along the airfoil is given. One must, however, be very aware that the choice of airfoil data directly influences the results from the BEM method. For certain airfoils a lot of experience has been gathered regarding appropriate corrections to be used in order to obtain good results, and, because of this, blade designers tend to be conservative in their choice of airfoils. With maturing CFD algorithms especially for the transition and turbulence models and more wind tunnel tests, the trend is now to use airfoils specially designed and dedicated to wind turbine blades, see e.g. [22].

### 2.1.5. Wind simulation

Besides airfoil data, also realistic spatial-temporal varying wind fields must be generated as input to an aeroelastic calculation of a wind turbine. As a minimum the simulated field must satisfy some statistical requirements, such as a specified power spectre and spatial coherence, see [23,24]. In this method, each velocity component is generated independently from the others, meaning that there is no guarantee for obtaining correct cross-correlations. In [25], a method ensuring this is developed.
on the basis of the linearized NS equations. In the future, wind fields are expected to be generated numerically from Large Eddy Simulations (LES) or Direct Numerical Simulations (DNS) of the NS equations for the flow on a landscape similar to the actual siting of a specific wind turbine.

2.2. Lifting line, panel and vortex models

In the present section, 3D inviscid aerodynamic models are reviewed. They have been developed in an attempt to obtain a more detailed description of the 3D flow that develops around a wind turbine. The fact that viscous effects are neglected is certainly restrictive as regards the usage of such models on wind turbines. However, they should be given the credit of contributing to a better understanding of dynamic inflow effects as well as the credit of providing a better insight into the overall flow development [11,12]. There have been attempts to introduce viscous effects using viscous–inviscid interaction techniques [26,27] but they have not yet reached the required maturity so as to become engineering tools, although they are 3D models that can be used in aeroelastic analyses.

2.2.1. Vortex methods

In vortex models the rotor blades, trailing and shed vorticity in the wake are represented by lifting lines or surfaces [28]. On the blades the vortex strength is determined from the bound circulation that stems from the amount of lift created locally by the flow past the blades. The trailing wake is generated by the spanwise variation of the bound circulation while the shed wake is generated by a temporal variation and ensures that the total circulation over each section along the blade remains constant in time. Knowing the strength and position of the vortices the induced velocity can be found in any point using the Biot–Savart law, see later. In some models (namely the lifting-line models) the bound circulation is found from airfoil data table-look up just as in the BEM method. The inflow is determined as the sum of the induced velocity, the blade velocity and the undisturbed wind velocity, see Fig. 1. The relationship between the bound circulation and the lift is denoted as the Kutta–Joukowski theorem (first part of Eq. (2.2.1)), and using this together with the definition of the lift coefficient (second part of Eq. (2.2.1)) a simple relationship between the bound circulation and the lift coefficient can be derived

\[ L = \rho V \Gamma = 1/2 \rho V^2 C_l \Rightarrow \Gamma = 1/2 V \Gamma, \]

(2.2.1)

Any velocity field can be decomposed in a solenoidal part and a rotational part as

\[ \mathbf{V} = \nabla \times \Psi + \nabla \Phi, \]

(2.2.2)

where \( \Psi \) is a vector potential and \( \Phi \) a scalar potential [29]. From Eq. (2.2.2) and the definition of vorticity a Poisson equation for the vector potential is derived

\[ \nabla^2 \Psi = -\omega. \]

(2.2.3)

In the absence of boundaries, \( \Psi \), can be expressed in convolution form as

\[ \Psi(x) = \frac{1}{4\pi} \int \frac{\omega'}{|x - x'|} \, d\text{Vol}, \]

(2.2.4)

where \( x \) denotes the point where the potential is computed, a prime denotes evaluation at the point of integration \( x' \) which is taken over the region where the vorticity is non-zero, designated by Vol. From its definition the resulting induced velocity field is deduced from the induction law of Biot–Savart

\[ \mathbf{w}(x) = -\frac{1}{4\pi} \int \frac{(x - x') \times \omega'}{|x - x'|^3} \, d\text{Vol}. \]

(2.2.5)

In its simplest form the wake from one blade is prescribed as a hub vortex plus a spiralling tip vortex or as a series of ring vortices. In this case the vortex system is assumed to consist of a number of line vortices with vorticity distribution

\[ \omega(x) = \Gamma \delta(x - x'), \]

(2.2.6)

where \( \Gamma \) is the circulation, \( \delta \) is the line Dirac delta function and \( x' \) is the curve defining the location of the vortex lines. Combining (2.2.5) and (2.2.6) results in the following line integral for the induced velocity field:

\[ \mathbf{w}(x) = -\frac{1}{4\pi} \int_{S} \frac{\Gamma}{|x - x'|^3} \times \frac{\partial x'}{\partial S} \, dS, \]

(2.2.7)

where \( S \) is the curve defining the vortex line and \( S' \) is the parametric variable along the curve.

Utilizing (2.2.7) simple vortex models can be derived to compute quite general flow fields about wind turbine rotors. The first example of a simple vortex model is probably the one due to Joukowski [30], who proposed to represent the tip vortices by an array of semi-infinite helical vortices with
constant pitch, see also [31]. In [32], a system of vortex rings was used to compute the flow past a heavily loaded wind turbine. It is remarkable that in spite of the simplicity of the model, it was possible to simulate the vortex ring/turbulent wake state with good accuracy, as compared to the empirical correction suggested by Glauert, see [6]. Further, a similar simple vortex model was used in [33] to calculate the relation between thrust and induced velocity at the rotor disc of a wind turbine, in order to validate basic features of the streamtube-momentum theory. The model includes effects of wake expansion, and, as in [32] simulates a rotor with an infinite number of blades, with the wake being described by vortex rings. From the model it was found that the axial induced velocities at the rotor disc are smaller than those determined from a uniform flow past the blades themselves can be found by applying a surface distribution of sources \( \sigma \) and dipoles \( \mu \) (Fig. 4). The background is Green’s theorem, which allows obtaining an integral representation of any potential flow field in terms of the singularity distribution [36,37].

\[
V(x; t) = V_0 + \nabla \phi(x; t);
\]

\[
\phi(x; t) = - \int_S \left( \frac{\sigma(t)}{4\pi|x-x'|} - \frac{\mu'(t) \cdot n' \cdot r}{4\pi|x-x'|^3} \right) dS',
\]

(2.2.8)

\( V_0 \) denotes a given (external) potential flow field possibly varying in time and space and \( \phi \) is the perturbation scalar potential. \( S \) stands for the active boundary of the flow and includes the solid boundaries of the flow \( S_B \) as well as the wake surfaces of all lifting components \( S_W \). In (2.2.8) \( \mu, \sigma \)

are defined as jumps of \( \phi \) and its normal derivative across \( S \): \( \mu = - [\partial \phi] \) and \( \sigma = [\partial_n \phi] \) with \( n \) defined as the unit normal vector pointing towards the flow. Source distributions are responsible for displacing the unperturbed flow so that the solid boundaries are shaped as flow surfaces and therefore are defined on \( S_B \). Dipoles are added so as to develop circulation into the flow to simulate lift. They are defined on \( S_W \) and the part of \( S_B \) referring to the lifting components. In fact a surface distribution of \( \mu \) is identified to minus the circulation around a closed circuit which cuts the surface on which \( \mu \) is defined at one point: \( \Gamma = -\mu \).

An important result given by Hess [36] states that the flow induced by a dipole distribution \( \mu \) defined on \( S_\mu \) is given by a generalization of the Biot–Savart law:

\[
\nabla \int_{S_\mu} \frac{\mu' \cdot (x - x')}{4\pi|x-x'|^3} dS'
\]

\[
= \int_{S_\mu} \left( \frac{\nabla' \mu \times n'}{4\pi|x-x'|^3} \right) (x - x') dS'
\]

\[
+ \frac{\mu' \cdot (x - x')}{4\pi|x-x'|^3} dS'
\]

(2.2.9)

where the line integral is taken along the boundary of \( S_\mu \) and \( s \) is the unit tangent vector to \( \partial S_\mu \) in the anticlockwise sense. If \( S_\mu \) is a closed surface the line integral vanishes, whereas if \( \mu \) is piecewise constant as in the vortex lattice method, the surface term will vanish leaving only the line term which corresponds to a closed-loop vortex filament present along all lines of \( \mu \) discontinuity on \( S_\mu \). The two terms on the RHS of (2.2.9) have the form as the Biot–Savart law (2.2.5). From this analogy, \( \gamma = \nabla \mu \times n \) is called surface vorticity and \( \mu \) line vorticity, which justifies the term vortex sheet for the wake of lifting bodies.

In potential theory a wake surface is the idealization of a shear layer in the limit of vanishing thickness. For an incompressible flow, the flow will exhibit a velocity jump: \( \left[ \frac{V}{w} \right]_W = -\nabla \mu_W \) while \( \left[ \frac{V}{w} \right]_W \cdot n = 0, \left[ \frac{p}{w} \right]_W = 0 \). Using Bernoulli’s equation, it follows that

\[
\frac{\left[ \frac{p}{w} \right]_W}{\rho} = \frac{\partial \mu_W}{\partial t} + V_{W,m} \cdot \left[ \frac{V}{w} \right]_W
\]

\[
= \frac{\partial \mu_W}{\partial t} + \left( V_{W,m} \cdot \nabla \right) \mu_W = 0,
\]

(2.2.10)

where \( V_{W,m} = (V_W + V_W^c)/2 \). Since, \( \Gamma = -\mu_W \), Kelvin’s theorem is obtained from (2.2.9) provided that \( S_W \) is a material surface moving with the mean...
flow $V_{W,m}$. Circulation will be materially conserved and therefore $\mu_W$ is identified with its value at $t = 0$. For a lifting problem, this means that $\mu_W$ is known from the history of the wing loading. Assuming that $S_W$ starts at the trailing edge of the wing, the generation of the wake can be viewed as a continuous release of vorticity in the free flow. The streak line of a point along the trailing edge will reveal the history of the loading of the specific wing section as indicated in Fig. 7.

The first model developed within the above context is Prandtl’s lifting-line theory, see e.g. [38]. It concerns a lifting body of vanishing chord (or else large aspect ratio), and thickness (Fig. 5). So $\sigma \equiv 0$ whereas $S_B$ becomes a line carrying the loading $\Gamma(y)$ (bound vorticity), which is the only unknown since the vorticity in the wake (trailing vorticity) is given by $\partial_y \Gamma(y)$. In Prandtl’s original model, $\Gamma(y)$ is determined from airfoil data as equation (2.2.1).

Then, as an introduction to lifting-surface theory, bound vorticity was placed along the $c/4$ line while along the $3c/4$ the non-penetration condition was applied in order to determine $\Gamma(y; t)$. The next step was to introduce the lifting body as a lifting surface. The most widely used model in this respect is the vortex-lattice model [39]. It consisted of dividing $S_B$ and $S_W$ into panels and defining on them piecewise constant $\mu$ distributions (Fig. 6). Then according to (2.2.9) the perturbation induced by the wing and its wake, is generated by a set of closed-loop vortex filaments each defined along the boundary of a panel. The dipole intensities on the wing can be determined by the non-penetration condition at the panel centres whereas along the trailing edge, see (2.2.10) $\mu = \mu_W$ to ensure zero loading locally. In the case of an unsteady flow, the loading $\Gamma_B$ will change so that the vorticity shed in the wake will also have a cross component $-\delta \Gamma_B/\partial t \cdot \delta t$, as indicated in Fig. 6.

Having determined $\mu$ it is possible to calculate the induced velocity and thus the angle of attack and finally the loads from an airfoil data table look-up. Another option frequently used in propeller applications, is to determine lift by integrating the pressure jump along the section. Then by considering that the lift force is perpendicular to the effective inflow direction the angle of attack is determined. In this case the pressure jump is obtained directly from Bernoulli’s equation over the section except at the leading edge where the geometrical singularity of a blade with no thickness makes it necessary to include the so-called suction force. In propeller applications where this concept was first introduced, the suction force is estimated by means of semi-empirical modelling [40]. Whenever used in wind energy applications, the suction force has been determined as $-\rho V \times \Gamma \delta l$, where $V$, $\Gamma$ are the local values at the leading edge and $\delta l$ represents the vector length along the leading edge line. In general, the two schemes give comparable results. It is difficult to clearly state which scheme is better to use, since deviations appear as the angle of attack increases so that a theoretical justification based on matched asymptotic expansions is difficult. Another point of concern, regarding both lifting theories, is the detail in which the flow can be recorded. The fact that the flow geometry is approximate suggests that only at some distance from the solid boundary the flow could be meaningful. Finally, for the same reason, viscous corrections based on boundary layer theory cannot be applied.

In order to overcome these difficulties, the exact geometry of the flow had to be included. This was done by Hess who first introduced the panel method.

![Fig. 5. The lifting-line model.](image-url)

![Fig. 6. The lifting-surface model.](image-url)
in its full form [41]. Now the panel grid is defined on the true solid boundary and a piecewise constant source distribution is introduced which can be determined by the no-penetration boundary condition. In order to account for lift dipole distributions are added. Because there is no kinematic condition to determine \( \mu \), Hess defined \( \mu \) to vary linearly along the contour of each section (Fig. 7): \( \mu(s, y; t) = s \cdot \Gamma(y; t)/L(y) \) and used (2.2.10) at the trailing edge as an extra condition for determining \( \Gamma(y; t) \) (Fig. 7). The resulting problem is non-linear and an iterative procedure must be used which penalizes the computational cost considerably, as compared to the lifting-surface model.

An important aspect of potential flow models concerns wake dynamics. As discussed earlier, the wake of a lifting body is a moving surface and \( S_W \) should be allowed to change in time. Regarded as a vortex sheet, the evolution of \( S_W \) in time will be subjected to convection and deformation. For example in the case of the vortex lattice method, each wake segment will conserve its intensity but its vector length \( \delta l \) will satisfy the following equation:

\[
\frac{d}{dt} \delta l = (\delta l \nabla) V. \tag{2.2.11}
\]

As time evolves, wake deformations will generate singularities such as intense roll-up along the wake extremities and crossings. In fact at finite time the flow will blow-up. In order to avoid blow-up, corrective actions must be taken. If the simulation retains the connectivity of the wake surface, the lines defining the wake must be smoothened regularly during the run time. Alternatively one can apply remeshing which consists in dividing the wake vortex segments so that they do not exceed a prescribed upper limit. Both schemes, however, require substantial bookkeeping and quite intense procedures. With panel methods this problem becomes more complicated because the wake will also contain a surface vorticity term. A completely different approach is to discard wake connectivity. Rehbach [42] was the first to note that for an incompressible flow, vorticity concentrations \( \delta \Omega \) of the type \( \omega \delta D \), \( \gamma \delta S \) and \( \Gamma \delta l \) behave similarly. Their kinematic analogy was already discussed with reference to (2.2.9). Dynamically they all satisfy the same evolution equation:

\[
\frac{D}{Dt} \delta \Omega \equiv \frac{\partial}{\partial t} \delta \Omega + (u \nabla) \delta \Omega = (\delta \Omega \nabla) u, \tag{2.2.12}
\]

where \( D/Dt \) denotes the total or material time derivative. So Rehbach integrated the wake vorticity into point vortices, which subsequently moved freely as fluid particles carrying vorticity. This procedure provides substantial flexibility in the evolution of the wake. He also introduced the concept of modifying the kernel of the Biot–Savart law in order to cancel the \( r^{-2} \) singularity. The theoretical justification of Rehbach’s method came later on leading to the development of the vortex blob method [43].

In vortex models, the wake structure can either be prescribed or computed as a part of the overall solution procedure. In a prescribed vortex technique, the position of the vortical elements is specified from measurements or semi-empirical rules. This makes the technique fast to use on a computer, but limits its range of application to more or less well-known steady flow situations. For unsteady flow situations and complicated wake structures, free wake analysis become necessary. A free wake method is more straightforward to understand and use, as the vortex elements are allowed to convect and deform freely under the action of the velocity field as in Eq. (2.2.12). The advantage of the method lies in its ability to calculate general flow cases, such as yawed wake structures and dynamic inflow. The disadvantage, on the other hand, is that the method is far more computing expensive than the prescribed wake method, since the Biot–Savart law has to be evaluated for each time step taken. Furthermore, free wake vortex methods tend to suffer from stability problems owing to the intrinsic singularity in induced velocities that appears when vortex elements are approaching each other. To a certain extent this problem can be remedied by introducing a vortex core model in which a cut-off parameter.

**Fig. 7.** The exact potential model of a wing.
models the inner viscous part of the vortex filament. In recent years, much effort in the development of models for helicopter rotor flow fields have been directed towards free-wake modelling using advanced pseudo-implicit relaxation schemes, in order to improve numerical efficiency and accuracy, e.g. [44,45].

To analyse wakes of horizontal axis wind turbines, prescribed wake models have been employed by e.g. [46–48]. Free vortex modelling techniques have been utilized by e.g. [49,50]. A special version of the free vortex wake methods is the method described in [51], where the wake modelling is taken care of by vortex particles or vortex blobs.

Recently, the model of [52] was employed in the NREL blind comparison exercise [137], and the main conclusion from this was that the quality of the input blade sectional aerodynamic data still represents the most central issue to obtaining high-quality predictions. Nevertheless, it is worth noticing that by introducing relaxing techniques in the wake evolution [53], it is nowadays possible to run a large number of revolutions which is of importance in aeroelasticity with reference to fatigue and stability analysis; see later.

An alternative to panel methods is offered by the Boundary Integral Equation Methods (BIEM). By assuming stagnant flow inside the blade, \( \mu = -\phi \) and \( \sigma = \partial_\phi \), the resulting integral equation is only weakly singular and so less expensive. Within the field of wind turbine aerodynamics, BIEMs have been applied by e.g. [54–56]; up to now, however, only in simple flow situations.

Vortex methods have been applied on wind turbine rotors particularly in order to better understand wake dynamics. The next and quite challenging step is to upgrade potential flow methods so as to also include viscous effects. Examples of applying viscous–inviscid coupling within the context of 3D boundary layer theory can be found in [26,57]. Also attempts to include separation were made in [58]. All these works, however, cannot be considered conclusive. There are several unresolved issues such as convergence at the inboard region where significant radial flow develops as a result of substantial separation, and the end conditions at the tip. The fact that current trends in wind turbine design indicate preference to pitch regulated machines could increase the interest in flow models based on inviscid considerations. Finally, another application of potential flow models is to use them in order to obtain far field conditions for RANS computations in view of reducing their computational cost [59].

### 2.3. Generalized actuator disc models

The actuator disc model is probably the oldest analytical tool for analysing rotor performance. In this model, the rotor is represented by a permeable disc that allows the flow to pass through the rotor, at the same time as it is subject to the influence of the surface forces. The ‘classical’ actuator disc model is based on conservation of mass, momentum and energy, and constitutes the main ingredient in the 1D momentum theory, as originally formulated by Rankine [60] and Froude [61]. Combining it with a blade-element analysis, we end up with the celebrated Blade-Element Momentum Technique [6]. In its general form, however, the actuator disc might as well be combined with the Euler or NS equations. Thus, as will be shown in the following, no physical restrictions have to be imposed on the kinematics of the flow.

A pioneering work in analysing heavily loaded propellers using a non-linear actuator disc model is found in [62]. Although no actual calculations were carried out, this work demonstrated the opportunities for employing the actuator disc on complicated configurations, such as ducted propellers and propellers with finite hubs. Later improvements, especially on the numerical treatment of the equations, are due to [63,64]. Recently, Conway [65,66] has developed further the analytical treatment of the method. Within wind turbine aerodynamics [67] developed a semi-analytical actuator cylinder model to describe the flow field about a vertical-axis wind turbine. A thorough review of ‘classical’ actuator disc models for rotors in general and for wind turbines in particular can be found in the dissertation [68]. Later developments of the method have mainly been directed towards the use of the NS or Euler equations.

In a numerical actuator disc model, the NS (or Euler) equations are typically solved by a second-order accurate finite difference/volume scheme, as in a usual CFD computation. However, the geometry of the blades and the viscous flow around the blades are not resolved. Instead the swept surface of the rotor is replaced by surface forces that act upon the incoming flow. This can either be implemented at a rate corresponding to the period-averaged mechanical work that the rotor extracts from the flow or by
using local instantaneous values of tabulated airfoil data.

In the simple case of an actuator disc with constant prescribed loading, various fundamental studies can easily be carried out. Comparisons with experiments have demonstrated that the method works well for axisymmetric flow conditions and can provide useful information regarding basic assumptions underlying the momentum approach [69–72], turbulent wake states occurring for heavily loaded rotors [73], and rotors subject to coning [74,75].

In Fig. 8, an example of how various wake states can be investigated by introducing a constantly loaded actuator disc into the axisymmetric NS equations is shown. By changing the thrust coefficient all types of flow states can be simulated, ranging from the wind turbine state through the chaotic wake state to the propeller state.

The generalized actuator disc method resembles the BEM method in the sense that the aerodynamic forces has to be determined from measured airfoil characteristics, corrected for 3D effects, using a blade-element approach. For airfoils subjected to temporal variations of the angle of attack, the dynamic response of the aerodynamic forces changes the static aerofoil data and dynamic stall models have to be included. However, corrections for 3D and unsteady effects are the same for generalized actuator disc models and the BEM model, hence the description of how to derive aerofoil data is the same as in Section 2.1.

In helicopter aerodynamics combined NS/actuator disc models have been applied by e.g. [76] who solved the flow about a helicopter employing a chimera grid technique in which the rotor was modelled as an actuator disk, and [77] who modelled a helicopter rotor using time-averaged momentum source terms in the momentum equations.

Computations of wind turbines employing numerical actuator disc models in combination with a blade-element approach have been carried out in e.g. [69,70,78,79] in order to study unsteady phenomena. Wakes from coned rotors have been studied by Madsen and Rasmussen [74], Mikkelsen et al. [75] and Masson et al. [78], rotors operating in enclosures such as wind tunnels or solar chimneys were computed by Hansen et al. [80], Phillips and Schaffarczyk [81] and Mikkelsen and Sørensen [82], and approximate models for yaw have been implemented by Mikkelsen and Sørensen [83] and Masson et al. [78]. Finally, techniques for employing the actuator disc model to study the wake interaction in wind farms and the influence of thermal stratification in the atmospheric boundary layer have been devised by Masson [84] and Ammara et al. [85].

The main limitation of the axisymmetric assumption is that the forces are distributed evenly along

![Fig. 8. Various wake states computed by actuator disc model with prescribed loading: (a) wind turbine state; (b) turbulent wake state; (c) vortex ring state; (d) hover state. Reproduced from [70,73].](image-url)
the actuator disc, hence the influence of the blades is taken as an integrated quantity in the azimuthal direction. To overcome this limitation, an extended 3D actuator disc model has recently been developed [86]. The model combines a 3D NS solver with a technique in which body forces are distributed radially along each of the rotor blades. Thus, the kinematics of the wake is determined by a full 3D NS simulation whereas the influence of the rotating blades on the flow field is included using tabulated airfoil data to represent the loading on each blade. As in the axisymmetric model, airfoil data and subsequent loading are determined iteratively by computing local angles of attack from the movement of the blades and the local flow field. The concept enables one to study in detail the dynamics of the wake and the tip vortices and their influence on the induced velocities in the rotor plane. A model following the same idea has recently been suggested by Leclerc and Masson [87]. A main motivation for developing such types of model is to be able to analyse and verify the validity of the basic assumptions that are employed in the simpler more practical engineering models. Reviews of the basic modelling of actuator disc and actuator line models can be found in [88], that also includes various examples of computations. Recently, another Ph.D. dissertation [89] carried out a simulation employing more than four million mesh points in order to study the structure of tip vortices. In the following we will give some examples of how the actuator disc/line technique may help in understanding basic features of wind turbine flows.

Computed iso-contours of vorticity for a three-bladed rotor with airfoil characteristics corresponding to the Tjæreborg wind turbine is shown in Fig. 9. In Fig. 10, a similar computation shows the formation of the trailing tip vortices. It is remarkable that the vortices are clearly visible more than 3 turns downstream. A new and interesting application of the actuator line model is to study the interaction between two or more turbines, especially for simulating park effects. In Fig. 11, the outcome of a computation in which the interaction between two wind turbines is simulated by replacing the two rotors by actuator lines with forces obtained from airfoil data is shown. Presently, this technique is used to investigate the effect of large wind farms including many up- and downstream wind turbines.

2.4. Navier–Stokes solvers

2.4.1. Introduction to computational rotor aerodynamics

The first applications of CFD to wings and rotor configurations were studied back in the late seventies and early eighties in connection with airplane wings and helicopter rotors [90–94] using...
potential flow solvers. To overcome some of the limitations of potential flow solvers, a shift towards unsteady Euler solvers were seen through the eighties [95–98]. When computing power allowed the solution of full Reynolds Averaged NS equations, the first helicopter rotor computations including viscous effects were published in the late eighties and early nineties [99–102].

In the late nineties, with the CFD solvers capable of handling viscous flow around rotors, application to wind turbine rotors became of practical interest. The first full NS computations of rotor aerodynamics was reported in the literature in the late nineties [103–107]. The European effort to apply NS solvers to rotor aerodynamics had been made possible through a series of National and European project through the nineties. The European projects dealing with development and application of the NS method to wind turbine rotor flows was the Viscous Effects on Wind turbine Blades (VISCWIND) from 1995 to 1997 [108], Viscous and Aeroelastic effects on Wind Turbine Blades, (VISCEL), 1998 to 2000 [109,110], and Wind Turbine Blade Aerodynamics and Aeroelasticity: Closing Knowledge Gaps, 2002 to 2004 [111–115].

2.4.2. Approaches

As a consequence of the origin of most CFD rotor codes from the aerospace industry and related research, many existing codes are solving the compressible NS equations and are intended for high-speed aerodynamics in the subsonic and transonic regime [116–120]. For the helicopter applications, where compressibility plays an important role, this is the natural choice. For wind turbine applications, however, the choice is not as obvious; one reason being the very low Mach numbers near the root of the rotor blades. As the flow here approaches the incompressible limit, Mach ~ 0.01 it is very difficult to solve the compressible flow equations. One remedy to improve their capability is the so-called preconditioning, that changes the eigenvalues of the system of the compressible flow equations by premultiplying the time derivatives by a matrix. On the other hand, the compressible solvers have many attractive features, among these the ease of implementation of overlapping and sliding meshes, application of high-order upwind schemes, and very well-developed solutions methods.

Another very popular method, especially in the US is the Artificial Compressibility Method [121,122], where an artificial sound speed is introduced to allow standard compressible solution methods and schemes to be applied for incompressible flows. In case of transient computations sub-iterations are taken within each time step to enforce incompressibility [122]. The method has several attractive features: Among these a similar ease of implementation of overlapping grids as the compressible codes. Overlapping grids are a necessity, to solve rotor/stator problems that are present when the rotor, tower and nacelle are all included in the computations. The main shortcoming of the method may be problems to enforce incompressibility in transient computations without the need for a huge amount of sub-iterations, and the problem of determining the optimum artificial compressibility parameter.

Due to the low Mach number encountered in wind turbine aerodynamics, an obvious choice is thus the incompressible NS equations. These methods are generally based on treating pressure as a primary variable [123–125]. Extensions to general curvilinear coordinates can be made along the lines of [126]. The method is not as easily extended to overlapping grids as the compressible and the artificial compressibility method, due to the elliptical pressure correction equation. But the method is well suited for solving the nearly incompressible problems often experienced in connection with wind energy. In connection with steady-state problems, the method can be accelerated using local time stepping, while the method using global time stepping still is well suited for transient computations.
2.4.3. Turbulence and transition

It is well known, that the NS equations cannot be directly solved for any of the cases of practical interest to wind turbines, and that some kind of turbulence modelling are needed. The standard approach to derive turbulence models is by time averaging the NS equation, resulting in the so-called Reynolds Averaged NS equations (RANS). Several different models have been used with good results for wind turbine applications, the most successful ones being the k-omega SST model of Menter [127], the Spalart–Allmaras model [128], and the Baldwin–Barth model [129]. The Baldwin–Lomax [130] model, often used in connection with helicopter and fix-wing applications, are not very well suited for wind turbine applications, where relatively high angles of attack are very common.

Several studies performed for stall controlled wind turbines, have shown that all RANS models lack the capability to model the stalled flow regime at high wind speeds. One possible way around this problem, the so-called Detached Eddy Simulation (DES) technique [131,132], has shown some promising results but still needs further validation. Additionally, the DES technique is much more computationally expensive than the standard RANS approach, as it needs much finer computational meshes and the computations needs to be computed with time accurate algorithms.

From experiments it is known that laminar/turbulent transition influences the flow over rotor blades for some cases. It has been demonstrated for 2D applications, that transition models can greatly improve the accuracy for cases where transition phenomena are important. Even though nearly all rotor studies so far have been computed assuming fully turbulent conditions, it is generally accepted that it is important to include laminar/turbulent transition to model the physics as close as possible [133,134]. Predicting transition in 3D is a much more complex task than dealing with 2D, and 3D transition is an active research field.

2.4.4. Geometry and grid generation

To compute a rotor using CFD, the first step is to obtain a digitized description of the blade geometry. Often the blade descriptions are given as spanwise sectional information, listing the airfoil section, the twist, the thickness, and the position with respect to the blade axis. Often the blades are highly twisted, and with a large taper in the spanwise direction.

Depending on the flow solver, different approaches to the mesh generation process exist: Cartesian cut cells, unstructured and structured and combinations of these. So far the majority of flow solvers applied to wind turbine research have utilized structured grids with hexahedral cells. In connection with structured grids there are several issues that need to be decided upon. Generally, the problem of making a high quality grid around a modern rotor cannot be handled by a single block configuration, but needs some kind of multi block mesh. These can either be conforming at the block boundaries, non-conforming or overlapping. The overlapping grids gives the highest degrees of freedom followed by the non-conforming and the conforming grids. Firstly, the grid needs to accurately resolve the blade shape, with good resolution of the leading edge and tip region. Secondly, the grids also need to resolve the regions around the blade with sufficient resolutions to capture the flow physics. As the Reynolds numbers are quite high, 1–6 million, the cells near the rotor blades become very thin, as the non-dimensional distance $y^+$ must be approximately 1 to resolve the laminar sub-layer and have accurate solutions. The mesh generation process calls for some degree of experience and grid refinement studies to verify that the grid is sufficient to resolve the desired physics. Also the grids need to extend far away from the rotor, in the order of several rotor diameters, to avoid disturbing the induced velocity field near the rotor blades. For axial flow conditions, the flow solvers often take advantage of the rotational periodicity of the rotor, solving only for a single blade using periodic conditions.

Using an unstructured flow solver with tetrahedral cells, the grid generation process is less cumbersome. But the problem of resolving very thin boundary layers using tetrahedral cells is well known, and it may be necessary to combine the solver with some kind of prismatic grids near the blade surface to avoid this problem. The use of unstructured flow solvers is not wide spread in connection with wind turbine aerodynamics, probably because of the limited geometrical complexity, and the strength of unstructured solvers mainly being their ability to cope with complex geometries.

2.4.5. Numerical issues

The codes typically used for wind turbines are of at least second-order accuracy in both time and space, often with an implicit time discretization.
scheme to loosen the time step restriction inherent to explicit methods. Typically, the viscous terms are discretized with central differences, while the convective terms are discretized with second- or third-order upwind schemes. To solve routinely for 5–10 million grid points, the solvers are often available in a parallelized version that allows for execution on several CPU’s in parallel. The rotating nature of the problem requires the use of either a moving frame including the non-inertial acceleration terms or a moving mesh option where so-called mesh fluxes must be included in the code. For a good overview of the numerical issues in connection with incompressible flow, see [135].

2.4.6. Application of CFD to wind turbine aerodynamics

The major part of wind turbine rotor computations performed until now has been focused on zero yaw rotor only configuration, where the nacelle and tower have been neglected, and the inflow to the rotor has been assumed to be steady without shear. This is, of course, a great simplification, but in many cases still a sufficiently good approximation. The effect of the tower on the rotor on an upwind turbine is comparable to other unsteady effects, such as incoming turbulence, time variations of the rotor and of the incoming flow. A simulation, working with a full turbine geometry has been tried [106]. This type of simulation is much more expensive, and needs some kind of sliding/overlapping mesh to accommodate the movement of the rotor with respect to the turbine tower and nacelle. Additionally, the simulation needs to be time accurate, and good resolution of the flow around the tower is needed to capture the tower wake far downstream of the turbine.

One of the first real proofs that CFD for wind turbine rotor applications can be useful came in connection with the blind comparison organized by the National Renewable Energy Laboratory in Boulder, Colorado in December 2000 [136–138]. Some of these results were later published in [139,140]. Here several wind turbine research groups were asked to compute a series of different operational conditions for the NREL Phase-VI turbine, corresponding to actual cases measured in the NASA Ames 80 \times 120 \text{ft} wind tunnel. When the results were made publicly available, it proved that one of the applied CFD codes were consistently reproducing the measured distribution of the aerodynamic forces along the blade span, even under highly 3D and extreme stall conditions.

The output extracted from typical CFD rotor computations are the low-speed shaft torque, or power production, and root flap moments, see Fig. 12. For modern full size commercial turbines, the power and root flap moment are typically the only available properties. Besides the quantities normally measured, the CFD simulations provide a huge amount of detailed information that can be used to provide more insight. The data typically extracted are the spanwise distributions of force coefficients, Fig. 13, the limiting streamlines on the blade surfaces, Fig. 14, and the sectional pressure distributions along the blade span, Fig. 15. For the NREL Phase-VI turbine, these detailed quantities can be compared to measurements, but this is not generally the case for commercially available turbines.

Another application of rotor CFD is the study of different aerodynamic details of the rotor, such as the blade tips, the design of the root section etc. Here, the CFD technique can be used to supply...
information, that the engineering methods are not capable of providing.

With the increase in computational power, it is today both possible and affordable to do yaw computations using CFD \[133,141–143\]. In contrast to the axial flow cases, the total rotor must be modelled in yaw simulations, thereby increasing the number of mesh points typically by a factor of three. Additionally, the azimuthally variation inherent in yaw simulations dictates a time accurate simulation, as no steady state solution can exist, thereby increasing the computing time severely. Typically, a yaw computation will be 10–20 times more expensive compared to steady state axial flow computations. Fig. 16 shows the normal and tangential force at the \( r/R = 30\% \) radius during one revolution at 60° yaw operation.

The release of the measurements on the NREL Phase-VI rotor has heavily influenced the CFD activities dealing with wind turbine rotor aerodynamics. This unique data set, with several well-documented cases, has given a new possibility to test details of state of the art CFD codes. In the years since the release of the measurements nearly half of all published CFD studies of wind turbine rotors deals with these measurements. Besides the references mentioned other places in this paper, the following studies use the NREL/NASA Ames measurements \[144–147\].

Recently DES simulations of rotors at realistic operational conditions have been attempted \[148\]. Again the NREL Phase-VI rotor was used to verify the model. The reason for this choice is, besides the availability of detailed measurements, the fact that the rotor has a limited aspect ratio, hence making the DES computations more affordable with respect to the number of grid cells. The mesh for this simulation consists of 15 million cells, compared to around 2 million cells for a standard RANS rotor computation. The fact that DES computations must be performed using time accurate computations, makes these types of simulations 20–40 times more
expensive than standard steady-state rotor computations. For modern-type rotors with large aspect ratios, the cell count would be even higher and the computations even more expensive. For the NREL Phase-VI rotor the improvement using the DES technique is very limited, but for other rotors where the RANS equations do not perform as well, DES may provide much more improvement. The use of pure Large Eddy Simulation, has been demonstrated in [149], where a computational grid of 300 million points is used to compute the initial transient of the development of a tip vortex.

2.4.7. Future

Today NS solvers are an important tool for analysing different wind turbine rotor configurations, and are used routinely along with measurements and other computational tools for development and investigation of wind turbines. NS solvers are especially well suited for detailed investigation of phenomena that cannot directly be accessed by simpler and less computationally expensive methods. Additionally NS methods can be used as a supplement to measurements, where they can be used both in the planning phase and to have a better interpretation of the actual physics in connection with the analysis of measurements.

Finally, one of the latest trends is to couple NS solvers to structural codes to perform full elastic computations of wind turbine rotors.

3. Structural modelling of a wind turbine

The main purpose of a structural model of a wind turbine is to be able to determine the temporal variation of the material loads in the various components. This is accomplished by calculating the dynamic response of the entire construction subject to the time-dependent load using an aerodynamic model, such as the BEM method. For offshore wind turbines also wave loads and perhaps ice loads on the bottom of the tower must be estimated. Two different and frequently used approaches to set up a dynamic structural model for a wind turbine are described in the next subsections.

3.1. Principle of virtual work and use of modal shape functions

The principle of virtual work is a method to set up the correct mass matrix, $M$, stiffness matrix, $K$, and damping matrix, $C$, for a discretized mechanical system as

$$M \ddot{x} + C \dot{x} + Kx = F_g,$$  \hspace{1cm} (3.1.1)

where $F_g$ denotes the generalized force vector associated with the external loads, $p$. Eq. (3.1.1) is of course nothing but Newton’s second law assuming linear stiffness and damping, and the method of virtual work is nothing but a method that helps setting this up for a multibody system and that is especially suited for a chain system. Knowing the loads and appropriate conditions for the velocities and the deformations, Eq. (3.1.1) can be solved for the accelerations wherefrom the velocities and deformations can be determined for the next time step. The number of elements in, $x$, is called the number of DOF, and the higher this number the more computational time is needed in each time step to solve the matrix system. Use of modal shape functions is a tool to reduce the number of DOF and thus reduce the size of the matrices to make the computations faster per time step. A deflection

Fig. 16. Azimuth variation of normal and tangential force coefficient, for the NREL Phase-VI turbine at the $r/R = 0.4$ section during a 60° yaw error at 15 m/s wind speed.
shape is here described as a linear combination of a few but physical realistic basis functions, which are often the deflection shapes corresponding to the lowest eigenfrequencies (eigenmodes). For a wind turbine such an approach is suited to describe the deflection of the tower and the rotor blades and the assumption is that the combination of the Power Spectral Density of the loads and the damping of the system do not excite the eigenmodes associated with higher frequencies. In the commercially available and widely used aeroelastic simulation tool, FLEX, see e.g. [150], only the first 3 or 4 (2 flapwise and 1 or 2 edgewise) eigenmodes are used for the blades. Results from this model are generally in good agreement with measurements, indicating the validity of the underlying assumption. First one has to decide on the DOF necessary to describe a realistic deformation of a wind turbine. For instance in FLEX4 17–20 DOFs are used for a three bladed wind turbine, with 3–4 DOFs per blade as described above, 4 DOFs for the deformation of the shaft (1 for torsion, 2 for the hinges just before the first bearing, with associated angular stiffness to describe bending, and 1 for pure rotation), 1 DOF to describe the tilt stiffness of the nacelle and, finally, 3 DOFs for the tower (1 for torsion, 1 for the first eigenmode in the direction of the rotor normal and 1 in the lateral direction).

The method of virtual work will only be briefly described. For a more rigorous explanation of the method the reader is referred to textbooks on dynamics of structures. The values in the vector describing the deformation of the construction, \( x_i \), are denoted the general coordinates. To each general coordinate is associated a deflection shape, \( u_i \), that describes the deformation of the construction when only \( x_i \) is different from zero and typically has a unit value. The element \( i \) in the generalized force corresponding to a small displacement in DOF number \( i \), \( dx_i \), is calculated such that the work done by the generalized force equals the work done on the construction by the external loads on the associated deflection shape.

\[
F_{g,i} = \int_S p \cdot u_i dS, \quad (3.1.2)
\]

where \( S \) denotes the entire system. Please note that the generalized force can be a moment and that the displacement can be angular. All loads must be included, i.e. also gravity and inertial loads such as Coriolis, centrifugal and gyroscopic loads. The nonlinear centrifugal stiffening can be modelled as equivalent loads calculated from the local centrifugal force and the actual deflection shape as shown in [151]. The elements in the mass matrix, \( m_{ij,k} \), can be evaluated as the generalized force from the inertia loads from an unit acceleration of DOF \( j \) for a unit displacement of DOF \( i \). The elements in the stiffness matrix, \( k_{ij,k} \), correspond to the generalized force from an external force field which keeps the system in equilibrium for a unit displacement in DOF \( j \) and which then is displaced \( x_i = \delta_{ij} \), where \( \delta_{ij} \) is Kroneckers delta. The elements in the damping matrix can be found similarly. For a chain system the method of virtual work as described here normally gives a full mass matrix and diagonal matrices for the stiffness and damping. For one blade rigidly clamped at the root (cantilever beam) it is relatively easy to estimate the lowest eigenmodes (first flapwise \( u_1^{1f} \), first edgewise \( u_1^{1e} \) and second flapwise \( u_2^{1f} \)) e.g. using an iterative method as described in [151]. The eigenmodes are normally described in a coordinate system aligned with the tip chord as e.g. shown in Fig. 1. It is practical to normalize the deflection shapes so that the tip deflection is unity. It is now assumed that any deflection can be described as a linear combination of these modes as

\[
u(x) = x_1 u_1^{1f}(x) + x_2 u_1^{1e}(x) + x_3 u_2^{1f}(x). \quad (3.1.3)
\]

The velocity and accelerations can be calculated, respectively, as

\[
u(t) = \dot{x}_1 u_1^{1f}(x) + \dot{x}_2 u_1^{1e}(x) + \dot{x}_3 u_2^{1f}(x) \quad (3.1.4)
\]

and

\[
u(t) = \ddot{x}_1 u_1^{1f}(x) + \ddot{x}_2 u_1^{1e}(x) + \ddot{x}_3 u_2^{1f}(x). \quad (3.1.5)
\]

The advantage of the method using generalized coordinates and modal shape functions is that the number of DOF in the dynamic system can be reduced to a relatively small number. Further, some high eigenfrequencies are filtered away, which is beneficial for the allowable time step, when computing deformations, \( x_i^{n+1} \), and velocities, \( \dot{x}_i^{n+1} \), at time \( t = (n+1)\Delta t \) from deformations, \( x_i^n \), velocities, \( \dot{x}_i^n \), and accelerations, \( \ddot{x}_i^n \), at time \( t = n\Delta t \). A good choice for the time integration scheme is the Runge–Kutta–Nyström method, which requires for stability reasons that the time step should resolve the highest eigenfrequency with 4 points; but for accuracy reasons 10 points is preferred. By reducing the highest eigenfrequency using a modal description of e.g. the blades not only
reduces the number of DOFs but also larger time steps can be taken.

3.2. FEM modelling of wind turbine components applying non-linear beam theory

Even though modal analysis offers a computationally effective way to analyse wind turbine dynamics, most of the recently developed aeroelastic codes use a full FEM approach [152], which allows a more complex deformation state of the wind turbine. The main features of such modelling procedures together with some aspects of non-linear beam theory are discussed in the present section.

The structural modelling of wind turbines is based on beam theory. For a 1D structure (beam) subjected to bending in two directions, torsion and axial tension, the formulation of the problem consists of two parts: (a) the elastic model which reduces the 3D structure of each component into a 1D structure concentrated along the elastic axis of the beam, and (b) the derivation of the dynamic equations. In classical beam theory (first order) the deformed position \( \mathbf{r} \) of any point initially at \( \xi_0 = (x, y, z)^T \) (Fig. 17) can be described by the displacement vector \( \mathbf{u} = (u, v, w, \theta)^T \) consisted of the two bending displacements \( u, w \) the torsion angle \( \theta \) and the tension \( v \):

\[
\mathbf{r} = \xi_0 + \mathbf{S}^0 \cdot \mathbf{u} + \mathbf{S}^1 \cdot \partial_\xi \mathbf{u},
\]

\[
\mathbf{S}^0 = \begin{bmatrix} 1 & 0 & 0 & z \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -x \end{bmatrix},
\]

\[
\mathbf{S}^1 = \begin{bmatrix} -x & 0 & -z & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\] (3.2.1)

Using Hooke’s law and assuming that shear stresses do not produce net torsion, the sectional elastic loads can be derived:

\[
\begin{align*}
F_y &= (EA)\partial_y v - (EA_x^*)\partial^2_y u - (EA^*)\partial^2_y w, \\
M_y &= (GI_y)\partial_y \theta, \\
M_x &= (EI_x^*)\partial^2_y u + (EI_{xy})\partial^2_y w - (EA^*)\partial_y v, \\
M_z &= -(EI_{zz})\partial^2_y u - (EI_{xz})\partial^2_y w + (EA^*)\partial_y v,
\end{align*}
\] (3.2.2)

where the terms in parentheses denote the averaged sectional properties of the structure. By considering the balance of loads and moments on of a beam

![Fig. 17. Kinematics and dynamics of a beam structure.](image-url)
element of length dy the beam equations are derived:

\[
\begin{align*}
\left( \int_A \mathbf{r} \cdot d\mathbf{m} \right) dy &= dF + \left( \int_A \mathbf{g} \cdot d\mathbf{m} \right) dy + \delta \mathbf{p} dy, \\
\left( \int_A \mathbf{r}_0 \times \mathbf{r} \cdot d\mathbf{m} \right) dy &= dM + d\mathbf{r} \times (F + dF) \\
+ \left( \int_A \mathbf{r}_0 \times \mathbf{g} \cdot d\mathbf{m} \right) dy + \mathbf{r}_a \times \delta \mathbf{p} dy,
\end{align*}
\]

(3.2.3)

where \( m \) denotes the mass per unit length, \( dF, dM \) are the net elastic loads on \( dy \), \( \mathbf{g} \) is the acceleration of gravity and \( \delta \mathbf{p} \) the sectional aerodynamic loads exerted at the aerodynamic centre \( \mathbf{r}_a = (x_a, 0, z_a) \) while

\[
r_0 = \{du + x + z\theta, dv + dy, dw + z - x\theta\}^T
\]

\[
\cong \{x + z\theta, dy, z - x\theta\}^T.
\]

In view of a more systematic and general framework for formulating dynamic equations, Hamilton’s principle has been also used [153,154].

By introducing (3.2.2) into (3.2.3) and (3.2.4) the beam dynamic equations are obtained:

\[
\int_A (S^0)^T \cdot \mathbf{r} \cdot d\mathbf{m} - \partial_y \int_A (S^1)^T \cdot \mathbf{r} \cdot d\mathbf{m} = \partial_y \left[ \mathbf{K}_{11} \cdot \partial_x \mathbf{u} \right]
\]

\[
+ \partial^2_{yy} \left[ \mathbf{K}_{22} \cdot \partial^2_{yy} \mathbf{u} \right] + \partial_y \left[ \mathbf{K}_{12} \cdot \partial^2_{yy} \mathbf{u} \right]
\]

\[
+ \partial^2_{yy} \left[ \mathbf{K}_{21} \cdot \partial_y \mathbf{u} \right] + \int_A (S^0)^T \cdot \mathbf{g} \cdot d\mathbf{m} + \mathbf{S}^a \cdot \delta \mathbf{p},
\]

(3.2.5)

\[
\mathbf{K}_{11} = \begin{bmatrix}
F_y & 0 & 0 & 0 \\
0 & EA & 0 & 0 \\
0 & 0 & F_y & 0 \\
0 & 0 & 0 & GI_f
\end{bmatrix},
\]

\[
\mathbf{K}_{22} = \begin{bmatrix}
EI_{zz} & 0 & EI_{xz} & 0 \\
0 & 0 & 0 & 0 \\
EI_{xz} & 0 & EI_{xx} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\mathbf{S}^a = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
za & 0 & -xa
\end{bmatrix},
\]

In case of flexible blades if large deformations are expected, as in the case of helicopter blades, second-order non-linear beam theory is to be used. The relevant developments originate from the work in [154]. The beam axis again lies along the \( y \)-axis but in the undeformed state of the beam while \( x \) and \( z \) denote the two bending directions of the beam. At its deformed state a local co-ordinate system \( O'\xi\eta\zeta \) is introduced which follows the pre-twist of the beam so that \( \xi \) and \( \zeta \) coincide with the local structural principle axes of the cross section (Fig. 18).

Then, (3.2.1) becomes

\[
\mathbf{r} = \begin{bmatrix}
u \\
v + w \\
w
\end{bmatrix} + \mathbf{E} \begin{bmatrix}
\xi \\
-\lambda \partial_y \theta
\end{bmatrix},
\]

(3.2.6)

where \( \lambda \) denotes the warping function of the cross section and \( \mathbf{E} \) is the transformation matrix between \( Oxyz \) and \( O'\xi\eta\zeta \). In [154], \( \mathbf{E} \) is given in terms of the elastic displacements up to second order. Next the strain tensor \( \varepsilon_{ij} \) defined through (3.2.6) is introduced in Hooke’s law in order to define the strain–displacements relations, which are used in the derivation of the resulting sectional elastic loads as in (3.2.2). Because they are derived in the \( O'\xi\eta\zeta \) system, before introducing them in (3.2.3) and (3.2.4) they are transformed into the \( Oxyz \) system using the
transformation matrix \( \mathbf{E} \). The final expressions are quite lengthy and the reader is referred to [154]. It is noted that compared to the classical beam model, second-order theory will produce fully complete stiffness matrices containing many non-linear terms.

The above non-linear beam model is a specific example amongst several models of varying complexity which have been gradually developed starting from the Timoshenko beam theory, by eliminating the ad hoc simplifying assumptions of the original model [155–158].

Regarding wind turbines, almost all structural models are based on classical beam theory [150,153,159–162]. This is partially due to the fact that wind turbine components are far more rigid compared to helicopter blades for which non-linear theory has been developed. Furthermore most of the difficulties in analysing wind turbine systems are generated by the aerodynamics and in particular the onset of stall, which is certainly less pronounced on helicopter rotors. Current designs are quite stiff so that non-linear beam modelling is not expected to change drastically the quality of predictions besides providing a sound basis for including the geometrical non-linearities [163]. However, if wind turbines become more flexible, it could become necessary to adopt such kind of structural modelling.

Regardless the details the beam equations are fourth order with respect to bending and second order with respect to tension and torsion. So for a FE approximation of the equations, \( C^1 \) shape functions are used for \( u, w \) and \( C^0 \) for \( v \) and \( \theta \). At the element level, \( u, w \) are approximated with third-order polynomials and the DOFs are the values and space derivatives at the end nodes while \( v \) and \( \theta \) are approximated with first-order polynomials and the DOFs again at the end nodes. In some models and certainly when the second-order beam theory is applied, second- and third-order polynomials are used, respectively, for \( v \) and \( \theta \). In this case the midpoint as well the points at 1/3 and 2/3 interior points are used as nodes. So within an element “\( e \)”,

\[
\mathbf{u}_e(y; t) = \mathbf{N}^e(y) \cdot \hat{\mathbf{u}}_e(t),
\] (3.2.7)

where \( \mathbf{N}^e(y) \) is the matrix containing the shape functions and \( \hat{\mathbf{u}}_e(t) \) the vector of the DOFs at the nodes of the elements [164]. As in Section 3.1 concerning the principle of virtual work, which in the FEM terminology is the Galerkin formulation of the problem is used to generate the discrete equations. With reference to (3.2.5) taken symbolically as \( \mathbf{Z}(\mathbf{u}, \hat{\mathbf{u}}, \ddot{\mathbf{u}}) = 0 \), for any admissible virtual

\[
\int_0^L \delta \mathbf{u}^T \cdot \mathbf{Z}(\mathbf{u}, \hat{\mathbf{u}}, \ddot{\mathbf{u}}) \, dy \\
\equiv \sum_e \int_0^{L_e} \delta \mathbf{u}_e^T \cdot \mathbf{Z}(\mathbf{u}_e, \hat{\mathbf{u}}_e, \ddot{\mathbf{u}}_e) \, dy = 0, \quad \forall \delta \mathbf{u}. \quad (3.2.8)
\]

Because \( \delta \mathbf{u}_e(y; t) = \mathbf{N}^e(y) \cdot \delta \hat{\mathbf{u}}_e(t) \), it follows that:

\[
\sum_e \int_0^{L_e} \mathbf{N}^e^T \cdot \mathbf{Z}(\mathbf{N}^e \hat{\mathbf{u}}_e, \mathbf{N}^e \hat{\mathbf{u}}_e, \mathbf{N}^e \ddot{\mathbf{u}}_e) \, dy = 0, \quad (3.2.9)
\]

which is a set of second-order ordinary equations in time with respect to \( \hat{\mathbf{u}}_e(t) \). Although \( \mathbf{Z} \) can contain non-linear terms, it can always be written in the usual form:

\[
\mathbf{M}(\mathbf{u}) \cdot \ddot{\hat{\mathbf{u}}} + \mathbf{C}(\mathbf{u}) \cdot \dot{\hat{\mathbf{u}}} + \mathbf{K}(\mathbf{u}) \cdot \hat{\mathbf{u}} = \mathbf{Q}(\mathbf{u}), \quad (3.2.10)
\]

where the mass, damping and stiffness matrices as well as the generalized loads in the RHS, in general will depend on the displacement field and its time and space derivatives (noted by a tilde).
The main components of a wind turbine are the blades, the drive-train and the tower (Fig. 19). They are all modelled as beam structures and typically the structural properties are assumed for each component to continuously vary along the corresponding elastic axis. However, localized properties can be added in the form of concentrated masses, dampers or springs. The gearbox (if present), the generator, the hub are usually added in this way. Other examples are the flexibility or damping characteristics of the yaw bearing or the pitch mechanism. The involvement of different body motions for each component in combination with the connections where loads and displacements are communicated from one component to the other, calls for a global formulation of the dynamic problem. To this end most works adopt a multi-body approach [165], which consists of considering each component separately subjected to appropriate boundary conditions, which fit the different components into the complete configuration. In such an approach the elastic DOF of each component are defined as local elastic displacements to which rigid body motions appear as a result of the time derivatives of the non-linearities of the connections are introduced using symbolic mathematics, such as Mathematica, carefully and systematically. Fortunately software will in response receive from the rest of the connected components their internal (or reaction) loads. This operation involves co-ordinate system transformations between the components. The set of kinematical DOF involved in the definition of \( \mathbf{p}_k \) and \( \mathbf{A}_k \) for all components is denoted collectively as \( \mathbf{q} \). So \( \mathbf{p}_k = \mathbf{p}_k(\mathbf{q}; t) \) and \( \mathbf{A}_k = \mathbf{A}_k(\mathbf{q}; t) \). \( \mathbf{p}_k \) is defined as a mixed series of translations and rotations, whereas \( \mathbf{A}_k \) is defined solely as a sequence of rotations. A typical example is shown in Fig. 20, where \( q_1-q_6 \) are the elastic DOF at the top of the tower, \( q_8-q_{13} \) are the elastic DOF of the drive train at the hub and \( q_7 \) and \( q_{14} \) are the yaw angle and the pitch of the blade, respectively. By introducing (3.2.11) into (3.2.5), the centrifugal and Coriolis terms of the inertia loads appear as a result of the time derivatives of \( \mathbf{A}_k \) while in the Hamilton’s principle approach they are produced automatically.

By combining the equations of all components the complete system of the dynamic equations for the wind turbine is obtained in the form of (3.2.10) with respect to an extended vector of DOF \( \mathbf{u}_{\text{ext}} = \{ \mathbf{\dot{u}}, \mathbf{q} \} \). By construction \( \mathbf{p}_k \) and \( \mathbf{A}_k \) will depend on unknown DOF while at the same time they correlate the state of the components in contact so the terms they generate are non-linear with respect to the unknown DOF \( \mathbf{u}_k \) and \( \mathbf{q} \). Linearization in this case is a tedious procedure which must be done carefully and systematically. Fortunately software using symbolic mathematics, such as Mathematica,
are available which are strongly recommended. The procedure is based on Taylor’s expansions of all variables and retaining of first-order terms. Alternatively, the extended version of (3.2.10) can be solved directly through an iterative procedure in each time step, which can use the linearized solution as a starting predictor.

4. Problems and solutions in wind turbine aeroelasticity

In the following section, the use of aeroelastic codes on wind turbine constructions including stability studies will be illuminated mainly through examples.

4.1. Aeroelastic stability

Wind turbines suffer from low structural damping which can become critical under certain operational conditions. Most problems appear on the blades that receive almost 100% of the loading. In particular the onset of stall plays a decisive role, not only on stall-regulated machines, but also on pitch-regulated ones around rated conditions. On the other hand, the blades are made of composite materials for which the knowledge on damping is substantially inferior compared to steel or concrete constructions. The damping of composite structures depends on the ambient temperature. So depending on the season, or the time during the day, the structural damping of the blades can decrease substantially. Furthermore, aging always degrades the damping of composite structures. There is a definite need to increase structural damping and significant effort has been put recently on its modelling in composite structures as well as in their appropriate design. This is one direction of research regarding the improvement of the stability characteristics of modern wind turbines; see [168] for a review of the current developments. Damping is required to suppress the onset of vibrations generated by the unsteady aerodynamic loads that interact with the wind turbine structure. First, the aeroelastic modelling of wind turbines is considered and then on this basis the aeroelastic stability problem is formulated and the relevant available methods are reviewed.

Aerodynamic modelling involves the calculation of the aerodynamic loads that also depend on the dynamics of the system. The flap and lag vibrations are interpreted by the incoming flow as a change in the effective angle of attack indicating strong aeroelastic coupling. All the currently available aeroelastic tools for wind turbines use 2D dynamic airfoil data models such as the Beddoes–Leishmann or the ONERA model. They provide aerodynamic coefficients as functions of the aerodynamic state variables that satisfy appropriate dynamic equations [17,169]. In either model the equations are 2D and so they are applied on a strip-by-strip basis, meaning that there is no interaction in the radial direction. This fits well with BEM modelling of the overall aeroelastic analysis. In fact the following presentation is referred to the BEM context, since most existing stability models are based on it. If a potential flow model is applied, then the attached part of lift can be associated to potential lift while the rest is simply added [163]. If the potential flow model is enhanced with a separation model there should be correlation on both aspects [58]. However, the latter has not been applied yet in aeroelastic computations.

By considering a blade section at a radial position as shown in Fig. 1 the local aerodynamic loads acting on this section of the blade can be written as follows, since the lift is perpendicular and the drag parallel to the relative wind:

\[ p_y = L \sin \phi - D \cos \phi, \]
\[ p_x = L \cos \phi + D \sin \phi, \]
\[ M_y = M, \]  

(4.1.1)

where \( L, D \) are the lift and drag forces, \( M \) is the pitching moment, \( \phi \) is the local flow angle with respect to the rotor plane. Once the induced velocity has been determined, the local effective incidence \( \alpha \) and relative flow velocity can be calculated as shown in Section 2.1. They will depend on the elastic deformations and the velocities, the rigid body motions and, if appropriate, orientation changes, as in pitch and yaw variations. In order to obtain a unified description of the coupled aeroelastic problem a so-called aeroelastic finite element is defined [170] which, in addition to the elastic DOF, the aerodynamic state variables added. For example in the ONERA model there will be in total eight additional DOF. Because the dynamic equations for the aerodynamic state variables only can be first order in time, in order to produce a consistent coupled system, they can be differentiated as in [171]. Therefore they can be assembled together with the dynamic equations in the form of (3.2.10) where the vector of the unknowns is further
extended to include the additional aerodynamic DOF. Another approach used in [172] is to introduce the modal expansion of the structural DOF at stand still, which better conditions the coupled system.

On the above basis it is possible to formulate the aeroelastic stability problem for wind turbines. Within the context of linear theory, stability boundaries are obtained by examining the evolution of small perturbations with respect to a steady state or a periodic solution acting as reference. To this end the original equations are linearized with respect to the reference state and the resulting system solved with respect to the fluctuations:

\[ \mathbf{M}(\mathbf{u}_{\text{ref}}) \cdot \ddot{\mathbf{u}} + \mathbf{C}(\mathbf{u}_{\text{ref}}) \cdot \dot{\mathbf{u}} + \mathbf{K}(\mathbf{u}_{\text{ref}}) \cdot \mathbf{u} = \mathbf{Q}, \quad (4.1.2) \]

where \( \mathbf{M}, \mathbf{C} \) and \( \mathbf{K} \) depend only on the reference solution. If the matrices do not depend on time, (4.1.2) is reformulated into a first-order system:

\[ \dot{\mathbf{y}} = \mathbf{D} \cdot \mathbf{y} + \mathbf{b}, \quad \mathbf{y} = \left\{ \hat{\mathbf{u}}, \ddot{\mathbf{u}} \right\}^T. \quad (4.1.3) \]

The eigenvalue analysis of \( \mathbf{D} \) provides the eigenfrequencies and damping of the system [173]. This is possible when the reference state corresponds to a steady solution as in the case of an isolated blade. However, when analysing the complete wind turbine configuration, due to the rotation of the blades, the reference state should correspond to a periodic equilibrium state with reference to the rotor speed, which is assumed constant. In this case the corresponding theoretical framework is Floquet’s theory, which for a large system is computationally heavy [174]. If the blades are identical and the number of blades \( N \geq 3 \), which is the most frequent case, it is possible by means of multi-blade transformation to eliminate the periodic coefficients in the coefficients of (4.1.2) and therefore be able to still use eigenvalue analysis [175]. In this context the non-linear equations of the system are integrated in time until such a periodic state is attained. In the case of an unstable situation, because the time domain response will contain significant components in all of the basic eigenfrequencies of the system, the reference state is obtained by truncating the response so as to retain only its 1 and \( N/\text{rev} \) components. To this end all DOF in the rotating system \( q_m, \ m = 1, \ldots, N \geq 3 \), are reformed: \( q_m = q_0 + q_c \cos \psi_m + q_s \sin \psi_m \), where \( \psi_m = \Omega t + (2\pi/N)(m - 1) \) is the corresponding azimuth location. Next the equations for the blade DOFs are rearranged by applying the operators \( (1/N)\sum_{m=1}^{N} (\cdots), (2/N)\sum_{m=1}^{N} (\cdots) \cos \psi_m \) and \( (2/N)\sum_{m=1}^{N} (\cdots) \sin \psi_m \). This is performed after the periodic solution has been obtained, over one period of rotation. The resulting equations will be in the non-rotating frame and contain only higher harmonics (3/rev and higher). By averaging over one period, the final constant coefficient system is obtained in the form of (4.1.3). For the blades \( y \) will contain: the corresponding \( q_0, q_c \) and \( q_s \) DOF and their time derivatives for both the purely structural and the aerodynamic DOF [171]. In the above approach it is worth noticing that the averaging procedure will force the reference state to appear in the equations. This is an important aspect when dealing with non-linear systems.

Stability of wind turbines has been considered systematically in Europe in the late 1990s [176], while today there is an on-going activity under the EU funded project STABCON. Several linear stability tools have been produced along the lines described above and good correlation has been obtained amongst the different codes, see [177] and the references cited. Inevitably linear theory is approximate so the results it produces are subject to cross checking. Clearly flutter measurements are difficult to obtain because of the involved risk, therefore it is indispensable to rely on theoretical developments which will be discussed in the next section.

4.2. Aeroelastic coupling: linear vs. non-linear formulations

Linearization of the aeroelastic equations certainly offers computational efficiency. However, it is not always appropriate. For the structure, linear theory requires that the deformations and displacements are small; an assumption not always valid. For the aerodynamics and its coupling with the structure, linearization will suppress some dependencies involved in (4.1.1) and therefore influence the estimation of the aerodynamic damping. In this connection the use of semi-empirical unsteady aerodynamic models introduces still unresolved uncertainties. Furthermore there are cases in which the multi-blade transformation is not applicable e.g. when blades are non-identical, or when the rotational speed varies.

Retaining fully the non-linearities of the aeroelastic problem has the following consequences: the structure is considered at its deformed state, the coupling conditions among the components are
taken in their full form, and the aerodynamic equations are not simplified. The last point is important with respect to the best possible estimation of the aerodynamic damping within the margins offered by the semi-empirical models used in the currently available aeroelastic tools. In this respect multi-body analysis, which has been adopted in most of the existing stability tools [161,178–181], offers this option by construction, provided that the structural model considers the construction at its deformed state and the code takes this into account in the definition of the local aerodynamics.

Under such conditions, the information of the stability characteristics of the wind turbine are contained in the time signals of the loads calculated through a non-linear time domain simulation. There exist two types of methodologies for retrieving this information: the one is based on the work-based approach [176], while the other is using signal-processing techniques. In the work-based method any mode shape is excited and loads are recorded until a steady periodic state has been reached. Then the aerodynamic damping is estimated by the work of the aerodynamic loads over the last period under the assumption that there is no energy interchange between the modes through the aerodynamic loads. In the signal processing approach, the system is harmonically excited for a finite duration. Then the transient response following the abrupt termination of the excitation is recorded wherefrom the aeroelastic damping and frequency of the specific mode are determined. There are two different methods, both well documented in the literature [182–184] that can be used: the moving block method and the method of Hilbert transform. The moving block method is a FFT based method, commonly used in rotorcraft applications [182,184]. The transient response amplitude is computed on a block of data using a FFT calculation. The block is then moved forward by a single point in time and the computation of the transient response amplitude is repeated. The linear fit for the slope of the natural logarithm of the sequence of response amplitudes in time (peak plot) provides the damping (Fig. 21). In the Hilbert method, the transient response \( \tilde{y}(t) \) and its Hilbert transform \( \tilde{y}(t) \) are used to define the envelope signal \( A(t) = \sqrt{y^2(t) + \tilde{y}^2(t)} \) [183,184]. For a transient response typical of a viscously damped system a line can be fitted to the logarithm of this envelope. The slope of this line, as in the case of the moving block method, gives the damping (Fig. 22). Comparing the two, it has been found that the Hilbert method has certain advantages over the moving block method. In particular the Hilbert damping analysis provides an estimate of the decaying envelope signal so it can be used in assessing the non-linear damping characteristics of a mode. Therefore it is not limited by the assumption of the viscous damping as the moving block method does. Moreover, experience has shown that the Hilbert method is more efficient in calculating the damping of spectrally close modes [163].

4.3. Examples of time simulations and instabilities

In this section the challenges in aeroelastic design of wind turbine rotors will be addressed and in particular the phenomenon of aeroelastic instability of wind turbine rotors will be explored by showing examples of simulation results on a wind turbine design.

Back from the beginning of development of modern wind turbine rotors in the late seventies there has been concern about the problems that could arise due to aeroelastic instability. In particular the use of the stall regulation principle was uncertain as it was foreseen that a flapwise instability (so-called stall flutter) would occur when operating in the stall region due to the negative slope of the \( C_L \) vs. \( \alpha \) curve. However, it turned out that the flapwise instability during operation in stall was not the most critical problem, but instead the edgewise instability, resulting in edgewise blade vibrations. The first experimental evidence of this instability was seen in the mid nineties and initiated considerable research activities in order to explore the phenomenon and provide practical solutions. Below, is mainly focused on this instability as this serves as a good example to illustrate the problem of aeroelastic instability in more general terms.

In the more recent wind turbine designs the regulation of the turbines has shifted from stall regulation to pitch control where the operational range for the flow over the blade moves to low angle of attack at high wind and thus away from the stall region. This has almost removed the instability associated with stall during operation but it is still so that the edgewise blade modes are aerodynamically low damped on the pitch regulated turbines.

A major instability problem on the modern turbines is seen when the rotor is parked or idling at very low RPMs at wind speeds above stop wind speed, which typically is around 25 m/s. Again it is typically edgewise dominated instability problems.
Fig. 21. Schematic description of the moving block damping analysis.
related to stall of the blade. A simulation example will be shown later to illustrate this problem.

Finally, the flutter instability will be addressed. It seems that the increased up scaling of the turbines has led to rotor and blade designs where the flutter speed is not so far away from normal operational speed. Also this instability will shortly be illustrated.

4.3.1. Edgewise blade vibration instability

As mentioned above the first experimental evidence of this instability was seen in the mid nineties on stall-regulated rotors with a diameter of 35–40 m. An example is presented in Fig. 23 and it is seen that the amplitude of the edgewise blade root moment (which at steady conditions vary sinusoidal with \(1p\) due to the gravity) increases 2–3 times due to instability during operation in stall. The experimental evidence of the edgewise instability led to considerable research on this subject and a major European project “Prediction of Dynamic Loads and Induced Vibrations in Stall” funded by the EU was carried out in the period from 1995–1998 [176].
The origin of the instability is simple. If a rotating airfoil section is harmonically translated along an axis $x_B$ and the direction of this axis $\theta_{RB}$ relative to the orientation $x_R$ of the rotor plane is varied, the aerodynamic damping coefficient for the section varies considerably, see Fig. 24 from [185]. For low $\theta_{RB}$ which means in-plane or edgewise vibration direction the damping coefficient is negative even at low wind speed. For vibrational directions close to $90^\circ$, which corresponds to out-of-plane or flapwise direction the damping coefficient is strongly dependent on the inflow wind speed. It is highly damped at low wind but close to zero or negative damped at high wind.

On a complete rotating blade the direction of vibration depends on the structural design of the blade as well as of the complete turbine structure. The movement of the tip section of the blade will now no longer necessarily be along a straight line. However, typically the tip section of the blade in the first flapwise mode will be on a path almost perpendicular to the rotor plane and in the 1st edgewise mode it will almost be in-plane. It is thus expected that the basic damping characteristics of

![Fig. 24. The aerodynamic damping coefficient $c_{xx,b}$ for a rotating airfoil section as function of vibration direction $\theta_{RB}$ and at three different inflow velocities, from [185].](image1)

![Fig. 25. Damping characteristics computed for a rotating, isolated blade using the linear aeroelastic stability tool HAWCStab [186].](image2)
the whole blade will be comparable with the characteristics of a single airfoil section. Using the linear stability tool HAWCStab the damping characteristics for a rotating blade can easily be derived and the computed damping characteristics for a rotating 19 m blade (500 kW rotor) corresponding to the blades of the rotor where the experimental instability was shown in Fig. 23, are shown in Fig. 25 from [186]. It is seen that the damping for the first flapwise mode varies from highly positive values at low wind speed and to values close to zero at 14 m/s and then increasing slightly at higher wind speeds. It should be noted that the influence of using unsteady blade section aerodynamics is shown (modelled with the Beddoes Leishman model in the present case) and that this has considerable influence on the aeroelastic damping. Generally, the unsteady aerodynamic effects increase the damping. The damping for the first edgewise mode is seen to decrease gradually with increasing wind from being slightly positive at low wind to slightly negative at high wind. Finally, it should also be noted that the damping shown is the total damping including the structural damping which in the present case is around 2% for the first flapwise mode and 3% for the first edgewise mode.

The final step in model complexity is now obtained by going to a full aeroelastic model of the turbine comprising the dynamics of the shaft, the nacelle and the tower. The aeroelastic stability results using HAWCStab is for the complete wind turbine, see e.g. Fig. 26 from [186], showing the aeroelastic damping for the first 10 mode shapes (numbered from lowest frequency). It is seen that two mode shapes are negatively damped at high wind speed and that one mode shape is close to zero and slightly negatively damped at the highest wind speed. For comparison, time simulations with the aeroelastic code HAWC [187,188] is performed on exactly the same turbine model at a wind speed of 8 and 16 m/s, respectively, in order to see if the instability at high wind speed predicted by the linear stability tool can be confirmed by the HAWC model, with non-linear aerodynamics. The computed edgewise blade root moment shown in Fig. 27 confirm the instability at high wind and compares very well with the measured instability shown in Fig. 23. It is thus demonstrated that the edgewise instability problem can be predicted in both time simulations and using linearized stability analysis. However, as will be demonstrated below, the edgewise vibration instability is much more complex than just edgewise vibrations of the individual blades but comprises the dynamic characteristics of the whole turbine. This understanding is vital for the development of design solutions preventing the instabilities.

In order to analyse the instability in more details the use of parallel aeroelastic time simulations on a turbine with a stiff structure is made. In this case the
loads on the different components will mainly reflect the direct load input from the external aerodynamic forces and from gravity forces. It should be mentioned that the simulations are performed at two average wind speeds of 8 and 16 m/s, respectively, and with a turbulence intensity of 12%.

A power spectral density analysis of the simulated edgewise blade root moment shows a distinct peak at 3.07 Hz which is close to the frequency of the first edgewise mode for a single, rotating blade which is 3.19 Hz, see Fig. 28. The response at 16 m/s of the flexible turbine at this frequency is seen to be several orders of magnitude bigger than the response of the stiff turbine indicating an instability situation. Somewhat the same tendency is seen at 8 m/s but with considerable less difference between the response of the stiff and flexible turbine indicating low damping but not an instability.

Analysing further the results of the stability analysis with HAWCStab as shown in Fig. 26 it can be derived from the results of the model that the three mode shapes that are negatively damped or very low damped at 16 m/s are:

- mode no. 1 with frequency 0.77 Hz and damping −6.05%,
- mode no. 3 with frequency 0.93 Hz and damping −4.33%,
- mode no. 7 with frequency 2.73 Hz and damping 1.59%.

![Fig. 27. Time simulation with the aeroelastic code HAWC on a 500 kW stall regulated rotor at two wind speeds, 8 and 16 m/s.](image1)

![Fig. 28. Comparison of power spectra of edgewise blade root moment for a flexible and a stiff turbine, respectively, at 8 and 16 m/s.](image2)
The numbering in HAWCStab of the modes is with increasing mode number with increasing frequency starting with the lowest frequency. Mode no. 1 and no. 3 are both modes where lateral tower bending is the main motion together with edgewise bending of the three blades and torsion of the main axis. The edgewise movement of the blades are all in phase and the difference between the two modes is the direction of the edgewise bending relative to the tower bending. Comparing again the results of the linear stability analysis with the time simulation results, the instability of the lateral tower bending dominated modes is confirmed by the spectra of the tower top and tower bottom lateral bending moments, Figs. 29 and 30. A distinct peak at 0.79 Hz is seen in both tower top and tower bottom spectra at 16 m/s and this peak cover probably also a considerable content at the 0.93 Hz, which was the frequency of mode no. 3.

It can be noticed that the HAWCStab results do not contain an unstable mode with a frequency around 3.07 Hz (the edgewise instability) as seen in the time simulation results. However, mode no. 7 is in fact the response on the non-rotating turbine structure from the edgewise vibrations at 3.07 Hz. The edgewise vibrations in the present case is a so-called backward whirlng, edgewise mode where there is a phase shift of 120°, between the movement of the blades. This whirling mode results in shaft and tower bending but with a frequency shifted 1 p

---

**Fig. 29.** Comparison of power spectra of tower top lateral moment for a flexible and a stiff turbine, respectively at 8 and 16 m/s.

**Fig. 30.** Comparison of power spectra of tower bottom lateral moment for a flexible and a stiff turbine, respectively, at 8 and 16 m/s.
\( p \) is the rotational frequency) up and down, respectively, compared with the frequency in the rotating system. As \( 1p \) in the present case is 0.45 Hz a peak in the spectra of tower bending at a frequency of around 3.07–0.45 Hz equal to 2.62 Hz is expected, which is also seen in Figs. 28 and 29. However, in the HAWCStab results a slightly higher frequency 2.73 Hz was seen for this mode.

The presented example of the edgewise blade vibration instability has shown that the low aerodynamic damping of an airfoil section undergoing a motion along a path close to the chordwise direction can cause instability of quite different modes on the turbine at the same time. The design challenges with the objective to minimize the influence of low aerodynamic damping or instability are thus big as e.g. changes in tower design could lead to e.g. edgewise vibrations on the rotor. A number of different methods to reduce the risk for edgewise vibrations on stall-regulated turbines have been investigated \([176,186]\) and comprises e.g. changes of the stalling characteristics of the airfoils by so-called “stall strips” as well as structural design of the blades to achieve optimal vibrational directions.

4.3.2. Instability problems of parked rotors

The turbines are normally designed to operate up to a certain maximum wind speed and above this wind speed the turbines are shut down. In the stopped conditions the rotors can be completely parked or they can idle with low rotational speed depending on the actual design of the control system. As part of the certification of the turbine it must be shown that the turbine can withstand the wind loads at extreme wind speed conditions with parked rotor and the wind coming from any direction in the case that the yawing system of the turbine is not functioning.

As an example, the simulation of a rotor, parked at a wind speed of 50 m/s and a yaw error of 60°, is shown in Fig. 31. The turbine is the same as above and the time track of the edgewise blade root moment indicates low damping as the amplitudes vary much in time. To the right in Fig. 31 is shown a power spectrum of the same signal and it shows that the blade mainly vibrates in two modes; mode 3 at a frequency of 0.93 Hz which was described above and the first edgewise blade frequency at about 3.19 Hz. For the present inflow direction the vibrations are not unstable but it has to be documented for all inflow directions.

4.3.3. Flutter instability

The last example of an instability is flutter. This is a well-known instability from the aircraft industry but has not yet been a problem on wind turbines and has probably not been seen on commercial turbines. However, with the increasing size of the blades it seems that the flutter speed decreases due to increasing structural flexibility of the blades and not least the torsional frequency decreases. Therefore, it is a good idea to include a flutter speed calculation in the design verification for e.g. 50 m blades and above.

Flutter involves two DOF of the blade; torsion and translation. The flutter speed decreases when the frequency of these two DOF approach each other. For 50 m blades the frequency of the first

![Fig. 31. Simulation of a parked rotor at 50 m/s and a yaw error of 60°. Time track of the edgewise blade root moment to the left and a power spectrum of the same signal to the right.](image)
flapwise mode is typically slightly below 1 Hz and the frequency of the first torsional mode will typically be in the range from 5 to 8 Hz. However, it has been seen that the flutter instability can occur by a coupling of the second flapwise mode and the first torsional mode of the blade and of course this is also a reason for a decreasing flutter speed on the bigger blades. Another important design parameter for the flutter instability is the centre of mass in the blade sections relative to the centre of the elastic axis. As the centre of mass moves away from the elastic axis in the direction of the trailing edge the flutter speed decreases.

An example of a simulated flutter instability is shown in Fig. 32. The blade does not represent an industrial blade but has selected structural parameters, which give a rather low flutter speed. The second flapwise mode for the blade is slightly above 2 Hz and the first torsional frequency is around 6 Hz. Further the centre of mass is positioned 10% of the chord length behind the elastic axis. The example is mainly intended to show the characteristics of the flutter instability, which is completely different from the instabilities that have been seen so far. In the simulation the rotor is free to speed up as the generator is disconnected. When the flutter speed is reached the instability develops within 1–2 s and this is completely different from e.g. the edgewise blade instability that can build up over a minute or more. This characteristic means the blade probably will be damaged immediately if the flutter speed is reached. A further consequence of this fast development is that the flutter can occur at a lower rotational speed of the rotor if there is a yaw error. This is demonstrated in the right part of Fig. 31 where there is a yaw error of 50°. Here, the rotational speed at flutter instability is 13% lower than if there is no yaw error and the wind speed is just 8 m/s. If the for example the yaw error of 50° occurred at around the stop wind speed of 25 m/s the rotational speed at the flutter instability would decrease even further.

5. Present and future developments of aeroelastic models

The improvement of existing aeroelastic codes and the development of new aeroelastic models are highly influenced by the design trends of new wind turbines and trends in the siting of the turbines because this determines the needs for new capabilities of the models.

5.1. Areas with influence on the development of aeroelastic models

5.1.1. Influence of up-scaling

So far the most important design trend has been the up-scaling which within the last 10 years has increased the maximum size of the mass produced turbines with a factor of 10 from about 500 kW with a rotor diameter of about 40 m to 5 MW turbines with a rotor diameter of 120 m. The newest turbines are all pitch controlled with variable speed and typically with some type of cyclic pitch for load alleviation. An accurate modelling of these new flexible turbines has increased the requirements to simulate complex coupled modes where the integrated flexibility of the tower, of the drive train and of the blades and in interaction with the control, is

Fig. 32. Example of flutter instability on a 50 m test blade where the rotor is free to speed up. To the left normal inflow whereas to the right there is a yaw error of 50°.
important. Also non-linear effects from considerable deflections of the blades and the tower are becoming important and the aeroelastic stability must be predicted with a good accuracy. Some typical frequencies for a 5 MW turbine could be:

- First tower bending frequency 0.2–0.25 Hz
- First flapwise frequency 0.8–1.0 Hz
- First edgewise frequency 0.9–1.1 Hz
- First blade torsional frequency 5.0–7.0 Hz

The decrease in the fundamental turbine frequencies arising from the up-scaling has the effect that excitation from the turbulent inflow has increased as the power spectrum of the turbulence peaks at around 0.05 Hz.

5.1.2. Siting of the turbines

The major part of all new turbines is placed in small or bigger groups and in some cases in wind farms with more than 100 turbines. This means that wake operation is part of the inflow conditions, which has to be simulated in the derivation of the total design loads of the turbines. Load alleviation by cyclic or individual pitch controls is important for such inflow conditions and the aeroelastic models should be suited for this.

Still the major part of turbines are set up on land but the offshore part will increase, at least if a 5–10 years interval is considered. On land, the best sites have been used in many countries and therefore more and more complex sites will be used. This means that rather complex inflow situations must be modelled as for example complex shear and non-uniform turbulence over the rotor disc.

5.1.3. Future trends in turbine design and siting

It seems that the speed of up-scaling might be slowed down for some years as the industry wants to be more focused on turbine reliability. One way to improve this is to have the individual turbine models on the market for more years than seen in the past.

However, the increasing offshore development will support the up-scaling tendency as the transport and erection of big turbine components is no longer a major problem. Further, the higher foundation costs offshore will also support the up-scaling trend.

Extending the possible offshore sites to deeper water, new foundation types, typical with multiple frames, will be developed and the aeroelastic codes should be able to handle these structures. A further step in the offshore development is the floating turbines, either as single floating turbines or more turbines on the same, floating frame.

Finally, the offshore market could have the effect that the two bladed turbine with a teetered downwind rotor again will be considered as an alternative to the three-bladed machines. The main barrier for the development of two bladed turbines has been the low frequency noise from the blade passage of the tower shadow. This will be of less importance offshore as the noise restrictions are much lower here.

5.2. Areas of development in present and new codes

Influenced by the continuously increasing requirements to the capabilities of the aeroelastic models as described above, existing models are being further developed and improved and new codes are written. Below some important areas of development will be discussed.

- non-linear structural dynamics,
- calculation of induction and its dynamics,
- wake operation,
- derivation of airfoil data for aeroelastic simulations,
- complex inflow,
- aerodynamics of parked rotors and
- offshore turbines including floating turbines.

5.2.1. Non-linear structural dynamics

So far almost all aeroelastic codes have contained the model assumptions of small deformations and rotations, but the increased flexibility of the turbines have made uncertainty about the validity of this assumption for the new designs. The influence of non-linear effects on the dynamic response of a turbine was treated in a recent paper [189] and different effects were considered. For example characteristics (frequency and damping) of the first edgewise mode will typically change as function of increased flapwise deflection because the edgewise bending will couple more and more with the torsional mode of the blade. Besides increased influence to non-linear effects from the flexibility of the turbine components the new foundations used offshore and in particular floating foundations can contribute significantly to big deflections and related non-linear effects.
To overcome this uncertainty about the importance of non-linear effects a few new non-linear aeroelastic codes have now been developed. Siemens wind power has developed a non-linear code BHawC based on co-rotaing elements [190] while the new non-linear code HAWC2 [191] developed by Risoe is based on multibody dynamics. The trend of including non-linear effects will continue in the future and codes based on modal expansion techniques will be developed to include more modes as e.g. the blade torsional mode.

5.2.2. Calculation of induction and its dynamics

The unsteady aerodynamic air loads are directly dependent on the computed induction at the same point on the blade through the angle of attack. Therefore, an accurate prediction of induction is of crucial importance.

As the rotor disc has become bigger and bigger relative to the scales in the turbulent inflow, increasing variation in inflow conditions over the rotor disc are seen. Wind shear contributes to this variation in inflow. The inflow variations results in likewise considerable variation in the loading expressed through the local thrust coefficient $C_{T_{\text{local}}}$ over the rotor disc and thus also in induction. The induction is therefore even in normal operation highly dynamic and the induction model should be adapted to these conditions. As an example, Fig. 33 (reproduced from [192]) shows the variation of the local thrust coefficient at the outer part of the blade on an 80 m diameter turbine in normal operation.

Another source contributing to the load variations and thus to the dynamic induction is the eigenmotion of the blades. This part is also increasing due to up-scaling and due to the more flexible designs. As an example a blade with a flapwise frequency of 1 Hz as mentioned above and vibrating with an amplitude of 1 m will experience a relative velocity component perpendicular to the chord of around $\pm 6 \text{ m/s}$ in maximum. This value can directly be compared with the variations in inflow and is thus considerable. Different developments to improve the computation of induction have been seen. One type of modelling is the coupling of a numerical actuator disc model to the structural part of the aeroelastic model. So far such models have mainly been used to verify the accuracy of BEM modelling. As an example the aeroelastic code HAWC at Risoe with a BEM-type induction modelling in the standard version, has been developed in another version HAWC-3D where the induction is computed with a 3D actuator disc model and used e.g. to investigate the accuracy of yaw modelling [142,193], see Fig. 34 from [142]. In the coming years aeroelastic codes coupled to an actuator disc or actuator line model for computation of induction will certainly be further developed.
but probably mainly used for research and for verification of simpler codes and not so much in the aeroelastic codes used by the industry due to the much longer simulation time.

However, besides a direct coupling of an actuator disc or line method to an aeroelastic code in order to compute the induction at each time step there are other ways to utilize the capabilities of an actuator disc model. One example is to compute the induction characteristics of a rotor within its wind speed operational interval with an actuator disc and then afterwards use these quasi-steady induction characteristics in aeroelastic simulations for the rotor. The BEM method in the new HAWC2 aeroelastic code developed at Risoe has been implemented in such a way that this method can be used. The method is shortly first to compute the local induction as function of local loading for the actual rotor in a number of calculation points \((n \times m)\) over the rotor with an actuator disc:

\[
a(r_n, \theta_m) = f(CT(r_n, \theta_m)).
\]

Afterwards, these induction functions are then used in the BEM method during time simulations on the rotor instead of the unique, single relationship from 1D momentum theory, \(C_T = 4a(1 - a)\).

In fact, one of the main deficiencies of the BEM method is that this induction formula concerns the whole rotor disc. Deviations are seen in regions with considerable radial variation in the loading, which means in the tip or root region. Another example where the momentum induction formula does not hold is if the rotor disc is not plane, e.g. coned rotors or rotors where the blades bend considerably.

5.2.3. Wake operation

Most turbines are now set up in clusters or wind farms and this means that such turbines in part of their lifetime operate in wake from one or more turbines. In the international standard for design of turbines IEC 61400-1 [2] the increased loading from operation in wake can be taken into account by using an increased, effective turbulence which depends on a number of parameters in the considered wind farm such as e.g. wind turbine spacing. However, it has turned out that in cases where more detailed knowledge of the increased loading on the different turbine components from wake operation is needed, another more detailed aeroelastic modelling is needed than just increasing the turbulence. One such aeroelastic modelling has been developed over the past few years [194,195]. The main components in the modelling is: (1)
computation of the wake deficit from the up-stream turbine with an actuator disc model; (2) meandering of this deficit from the big scales in the turbulence and (3) additional turbulence within the wake. One of the advantages of this new modelling compared to the method using an effective turbulence is that both the mean yaw loads as well as the yaw dynamics compare well with measurement. Fig. 35 from [194] shows the loads as function of wind direction and full wake operation at a direction of around 210°. The highest mean yaw loads occur in half wake operation at around 195° and 220°. Recently, the wake resulting from the interaction of several turbines in a row was computed in [196] by combining large eddy simulation of the NS equations with the actuator line methodology.

It is expected that the more detailed aeroelastic modelling of wake operation will be developed considerably in the coming years because such modelling is necessary for the development of advanced control algorithms adapted for load reduction in wakes. The development of the new detailed models can be in the direction like the model described above but also using actuator line models or full 3D rotor models to compute the wake characteristics, which can be fed into an aeroelastic simulation.

5.2.4. Derivation of airfoil data for aeroelastic simulations

The airfoil data used in aerodynamic and aeroelastic simulations are of crucial importance for the
accuracy of such simulations. This was clearly seen in the result of the NREL blind test [137] where the difference in the airfoil data used in the individual simulation models probably were the most significant factor for the big variation between the output of the different simulations. However, it should be noted that this result is not a representative indicator of the uncertainty in the industry on aerodynamic and aeroelastic results on new turbines as considerable experience has been learned from older rotor designs in order to adapt airfoil data sets so that computed rotor power and loads compare well with measurements. The drawback is that this introduces conservatism in the design and for example can hinder the use of new airfoils. Therefore, methods and models to derive or correct airfoil data to be used in aeroelastic simulations have been a key issue for many years in the wind energy research community and will also be in the near future. 2D wind tunnel airfoil data has so far been the common starting point to set up an airfoil data set for simulations. In order to correct for so-called 3D flow effects and rotational effects some kind of correction is typically applied [19–21,197]. The corrections reflect the increased lift and drag that was measured on different rotors in field rotor measurements performed 10–15 years ago [198,199]. Full 3D CFD rotor computations later confirmed clearly the tendencies of the 3D flow effects and rotational effects in the form of increased lift in the blade root region but also increased drag [137–139].

With the possibility of running full 3D simulations on a rotor a new source for derivation of 3D airfoil data sets for input in aerodynamic and aeroelastic codes has emerged. Methods to extract the airfoil data from 3D rotor computations have been developed [200,201]. One example of such a derived airfoil data set is shown in Fig. 36 from [202], where a considerable increased maximum lift on the inboard stations are seen. Within the next few years it is expected that a considerable effort will be on further development of the methods to extract airfoil data from 2D wind tunnel data as well as 3D CFD rotor data. So far the CFD computations have mainly been used to extract quasi-steady data but they will in the future also be used to tune the parameters in the dynamic stall models.

5.2.5. Complex inflow

Wind turbines are often set up in so-called complex terrain, which means some kind of mountainous terrain because the wind potential is good at such places. However, the consequence is that the inflow conditions can vary considerably from turbine to turbine due to local variations of the terrain. As a result extreme wind shear has been seen to occur over the rotor disc of turbines placed in such terrain and likewise the turbulence intensity can be extremely high.

The fast development of CFD codes has now made it possible to simulate the flow in details over the terrain spanned by a wind farm in such complex terrain. The flow data in the form of wind shear and turbulence can be used as input in aeroelastic simulations. There is thus a basis for a simulation tool to micro site turbines not only with respect to energy production but also loads is
expected. Thus type of modelling is certainly of high interest for the wind turbine industry because considerable costs to repair turbines due to extreme inflow conditions have been experienced in the past.

5.2.6. Aerodynamics of parked rotors

It is a common procedure to shut the turbines down at high wind, typically at wind speeds in the range from 20 to 25 m/s. Ultimate loads on parts of the wind turbine can occur during standstill conditions at very high wind speed whereas ultimate loads on other components will occur during operation below the stop wind speed.

The aerolelastic codes are therefore also used to compute the turbine response during standstill conditions and it is particularly the computation of the aerodynamic loads on the blades that is uncertain. This is due to the requirement that the loads on the parked rotor shall be investigated with the wind from all directions. The models must therefore be able to compute aerodynamic loads for angle of attacks in the complete range from $-180^\circ$ to $180^\circ$. Furthermore the inflow is turbulent so that it is highly unsteady aerodynamic loads and a severe complication is that some inflow conditions often lead to instability with the blade vibrating in a mode with negative aerodynamic damping. In a recent EU funded project “KNOWBLADE” different CFD codes were used to explore some basic characteristics of the blade standstill aerodynamics [115]. Because of the complexity of the aerodynamics it is also believed that the use of CFD will be the main path to explore the standstill aerodynamics in the coming years.

5.2.7. Offshore turbines including floating turbines

Presently, there are considerable research and development efforts on adapting aeroelastic models to simulate offshore wind turbines, mounted on a sub-structure standing on the sea-bed, see Fig. 37 from [203], or on a floating foundation.

Different approaches are seen in this development. One line is characterized by a complete integration of the standard aeroelastic model of the turbine with a hydroelastic model of the submerged supporting structure including the hydrodynamic loads, the wave loading and the soil forces on the part of the support structure in the sea bed [190,191]. In both these models the complete structure is described with finite elements and with engineering sub-models for computation of the hydrodynamic loads.

Another line of modelling is to couple a “standard” aeroelastic model to a separate module simulating the supporting structure, including wave loads and hydro loads. This approach has been presented by Repower [204] where the Flex5 aeroelastic code is coupled with a standard package ASAS for computations of wave loads on off shore structures. In the third development line the main idea is also to use a “standard” aeroelastic code and then couple it to a super element of the foundation. This approach has been presented by Vestas [203] where the foundation is modelled with a finite element model but then decomposed to a super element using the Craig–Bampton sub-structuring scheme. Considerable efforts on further developments of the aeroelastic models for off shore applications will be seen within the next years and with more emphasis on deeper water installations.

![Fig. 37. Different support structure concepts from [203].](image-url)
6. Discussion

On a wind turbine there is a strong coupling between the aerodynamic loads and the time-dependent structural behaviour of the construction. Statically a blade might change its twist and thus the angle of attack when deflected. But also the angles of attack are changed when the blades have a velocity relative to the fixed ground. For instance if the tower is moving upstream and everything else is stiff it will be felt by the blades as an increased wind speed and thus higher angles of attack will be present along the blades. The aerodynamic response will depend on how the lift and drag vary with the angle of attack. In stall the lift will decrease yielding a possible instability, whereas for attached flow the lift will increase and thus creating a higher load seen by the blade opposite the movement indicating stability. On a real wind turbine not only the tower is vibrating but all other components as well, which directly feeds back to the angle of attacks and thus the loads that again alter the motion. To simulate the aeroelastic response of a wind turbine, a non-steady structural model including the inertia must be made. Two methods, i.e. the method of virtual work applied on modal shape functions and the FEM, are addressed. To evaluate the aerodynamic loads, the BEM is still the most widely used due to its simplicity and computing efficiency. To obtain realistic results some engineering adds-on are necessary such as the Dynamic Wake, Dynamic Stall and a yaw model, which are therefore thoroughly described in this paper. A BEM method relies, however, on airfoil data and the results are therefore no better than the input. To avoid the uncertainties of the engineering adds-on the Actuator Line model can be used, since this method resolves the physics of these models through the NS equations. However, the actual blades are not resolved and to estimate the aerodynamic lift and drag airfoil data are still needed. To avoid airfoil data one needs to solve the NS equations and resolve the blades and the possible boundary layers. This is extremely computationally costly and therefore this model is not likely to replace the BEM method in the near future. However, the method can be used to extract airfoil data that can be used in the less advanced models. It is expected that the Actuator Line model very soon will replace the BEM method, since it contains less empirics. With the AL model the wake is also a part of the solution and therefore the effect from this wake on a wind turbine placed further downstream can be calculated. These simulations are becoming very important as wind turbines are grouped in large (offshore) wind parks. The inviscid flow models are included in this paper mainly for historical reasons, since they have played an important role in determining basic physical features. This type of model was e.g. used to calibrate the time constants in the Dynamic Wake model. Also a few Potential flow models exist that model the induced velocities and the wake behind a wind turbine by shedding vortex blobs from the blade surface. This model is similar to the AL model and may also in some aeroelastic codes replace the BEM method.

References

[1] Last og sikkerhed for vindmøllekonstruktioner, DANSK STANDARD DS 472 [in Danish].


