

# The physics of a stove-top espresso machine

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The operation of a common type of stove-top espresso machine is analyzed to determine the initial fill conditions required to have the coffee extracted in the optimum temperature range of 90 °C–95 °C. By using the gas laws and some benchtop experiments, it is shown that the pressure in the vessel increases much more slowly than does the saturated vapor pressure of water at the temperature of the vessel. For any given final temperature, the volume of coffee that can be extracted is linearly proportional to the initial volume of air in the pressure vessel; that is, the higher the fill level, the smaller the volume of coffee that can be extracted. It is also shown that for typical operating conditions for which the water is initially at room temperature, half of the coffee is extracted when the water temperature is below 70 °C, which is much less than the desirable temperature, and that hotter coffee extraction temperatures will result if the water is preheated to about 70 °C before the pressure vessel is sealed and at least 100 ml of air space is left in the vessel. Experiments confirming the analysis use easily obtained equipment and are appropriate for undergraduate laboratory work, with the added attraction that students can enjoy consuming the results of the experiments. © 2008 American Association of Physics Teachers.

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## I. INTRODUCTION

Espresso coffee, which is characterized by forcing water quickly (around 1 ml/s) through a ground-coffee plug at elevated pressure and temperature, can be made in many different ways. A study of the chemistry of coffee making<sup>1</sup> suggests that the flavors considered most desirable are extracted when the temperature of the water is about 93 °C, and the pressure is sufficient to allow the volume of water for a single “shot” (30–50 ml) to flow through the plug in about 30 s or less. If the water is hotter than 95 °C, the coffee tends to taste burnt, and if considerably less than 95 °C, some of the desirable flavors are not extracted. The combination of pressure and temperature differentiates espresso-type coffee from coffee brewed by common household methods such as a French press (plunger) or filter coffee, in which hot, often boiling, water is poured on the grounds, which are then separated by various methods of filtering after several minutes. Commercial and high-level espresso machines have independent controls over the pressure and temperature, by separately heating the water and then forcing it through the plug via a pump.

There is another common type of espresso coffee maker common in Europe (see Figs. 1 and 2), in which water is heated in a pressure vessel, and then forced through the plug at a pressure that depends on the temperature of the water in the pressure vessel. This type of espresso machine and its variants will be examined in this article.

## II. BASIC OPERATION

In this type of coffee maker, commonly known as a “moka,” the pressure vessel is partly filled with water, sealed at atmospheric pressure  $P_A$  and temperature  $T_i$ , and heated on a stove top. The increased pressure above the water displaces heated water from the pressure vessel through the coffee plug to extract the coffee, which collects in the top container. For typical stove-top heating conditions, and when the pressure vessel is typically half-full with water, coffee emerges from the spout after several minutes. Flow continues until

the water level falls below the lower end of the bottom tube, at which time steam will be forced through the plug and appear in the upper tube. Alternatively, if the water in the vessel is heated to a fixed final temperature, then the volume extracted depends on this temperature and the fill conditions. Although the latter method is preferable because the coffee extraction can be restricted to a desired temperature range, this method is not commonly used because most instruction manuals suggest that boiling the water is necessary for coffee to be produced, or that heating should continue until steam emerges from the upper tube. The latter is suggested as the normal mode of operation in a recent article.<sup>2</sup> A web search on “moka, coffee, and physics” shows many references on the need to boil the water and that the pressure inside is the saturated vapor pressure of water. In this article, it will be shown that boiling the water is not only unnecessary, but in many cases undesirable, and that the physics is not as simple as might first appear.

In the analysis it is assumed that the volume flow rate  $Q$  is proportional to the pressure drop across the plug:

$$Q = \alpha(P_{\text{tot}} - P_A), \quad (1)$$

where  $P_{\text{tot}}$  is the total pressure in the vessel,  $P_A$  is the atmospheric pressure, and  $\alpha$  is a fluid conductance, which depends on the cross-sectional area, the depth, and the degree of compaction of the coffee plug, as well as the effective viscosity of the water through the plug. The difference  $P_{\text{tot}} - P_A$  will be called the overpressure, to distinguish it from the pressure  $P_{\text{tot}}$ . The assumption that  $\alpha$  is a constant is convenient and is used to describe laminar flow through porous materials such as sand. Equation (1) is known as Darcy’s law.<sup>3</sup> For low velocity flow through the finely ground particles of the coffee plug the Reynolds number is considerably less than unity, so laminar flow conditions will occur. It is unlikely that  $\alpha$  remains constant during the coffee extraction process, because flow through the initially dry plug at cool temperatures can be very different from the flow that occurs later when the water is hotter and the grounds have absorbed water and swollen to form a much tighter mass. Despite the



Fig. 1. The stove-top machine that is the basis of this study.

simplifications involved,  $\alpha$  will be assumed to be constant, and the value used is that determined by measurements made at the end of the extraction process (see Sec. IV). Thus the real water flow will most likely be greater early in the extraction process than is calculated here, a fact that reinforces the conclusions reached. The quantitative and qualitative consequences of  $\alpha$  not being constant, or having a value different from that used, are discussed in Secs. III and IV, and the Appendix.

In using Eq. (1) to describe the flow, the hydrostatic pressure in the central exit tube connecting the pressure vessel to the coffee plug (see Fig. 2) has been ignored, because it is of

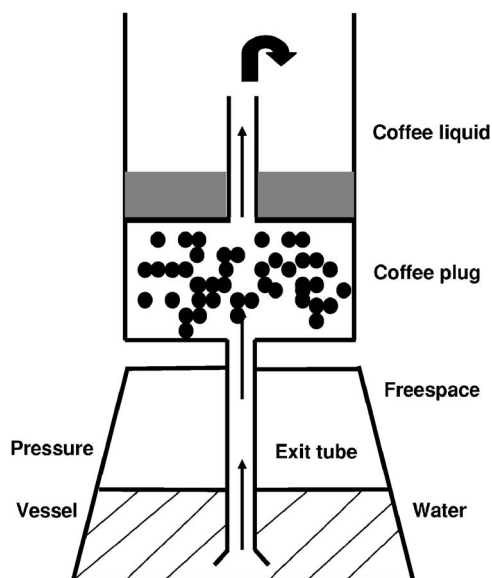


Fig. 2. A schematic of the basic operation of the machine.

the order of 1 kPa, much less than the pressure in the vessel when it is delivering coffee. We also ignore the effects of the changing solubility of air in water with temperature, because these effects are at most a 2% correction in going from 20 °C to 100 °C if there are approximately equal initial volumes of water and air in the vessel. The corrections would become larger as the initial volume of water exceeds the air volume, but they never become significant in situations of practical interest.

If the pressure vessel is at atmospheric pressure  $P_A$  and absolute temperature  $T_i$  when initially sealed and then warmed to temperature  $T$ , two effects come into play. The first effect is due to the increase in vapor pressure of water as it is heated. The simplest description of the relation between the saturated vapor pressures at temperatures  $T_i$  and  $T$  is derived by integrating the Clausius–Clapeyron equation<sup>4</sup> to give

$$P_{\text{sat}}(T) = P_{\text{sat}}(T_i) \exp \left[ \frac{LM}{R} \left( \frac{1}{T_i} - \frac{1}{T} \right) \right], \quad (2)$$

where  $L$  is the latent heat of vaporization,  $M$  is the molecular weight of water, and  $R$  is the universal gas constant. Equation (2) assumes that the water vapor acts as an ideal gas and the latent heat is constant over the entire temperature range, and is accurate to about 5% over the temperature range from 20 °C to 100 °C. For this reason, the calculations described later use the more accurate Groff–Gratch<sup>5</sup> formulation and Eq. (2) is used only in the Appendix where an analytic formulation is given.

The second effect comes from the change of the partial pressure of the air in the vessel (that is, the nonwater-vapor molecules trapped when the vessel is sealed). As water is forced out of the vessel, the volume available for the air increases. If we assume that the air behaves as an ideal gas, then as the air moves from volume  $V_i$  at  $T_i$  to volume  $V$  at temperature  $T$ , the air partial pressure  $P_{\text{air}}(T_i)$  changes to

$$P_{\text{air}}(T) = \frac{P_{\text{air}}(T_i) V_i T}{V T_i}. \quad (3)$$

Because the relative volume change of  $V/V_i$  is generally much greater than the relative absolute temperature change  $T/T_i$ , the partial pressure of the air decreases substantially as the coffee-making operation proceeds. For example, if there is 100 ml of space above the water when the vessel is sealed, and this space is filled by air at close to one atmosphere, then by the time 100 ml of coffee is extracted, the partial pressure of the air will have almost halved, causing the pressure to be almost 50 kPa less than might have been expected. This effect gives rise to some of the counterintuitive effects noted later; this aspect of the physics has been neglected in previous descriptions of the physics of these devices.<sup>2,6</sup>

The total pressure at any time is given by

$$P_{\text{tot}} = \frac{P_{\text{air}}(T_i) V_i T}{V T_i} + P_{\text{sat}}(T). \quad (4)$$

The pressure  $P_{\text{air}}(T_i)$  is determined from the conditions when the vessel is sealed, at which time

$$P_A = P_{\text{air}}(T_i) + P_{\text{sat}}(T_i), \quad (5)$$

so that

$$P_{\text{tot}} = \frac{[P_A - P_{\text{sat}}(T_i)]V_i T}{VT_i} + P_{\text{sat}}(T). \quad (6)$$

Equation (6) needs to be coupled with the flow equation, because the air volume  $V$  depends on how much water has been displaced. The rate of change of  $V$  is the water flow rate so that

$$\frac{dV}{dt} = Q = \alpha[P_{\text{tot}} - P_A]. \quad (7)$$

We substitute for  $P_{\text{tot}}$  from Eq. (6) and find

$$\frac{dV}{dt} = \alpha \left[ P_{\text{sat}}(T) + P_A \left( \frac{V_i T}{VT_i} - 1 \right) - \frac{P_{\text{sat}}(T_i) V_i T}{VT_i} \right], \quad (8)$$

where the temperature  $T$  is an increasing function of time. Equation (8) cannot be integrated analytically, even if  $T$  is a linear function of time, so it has been integrated numerically in the following examples. Equation (8) is no longer valid when the water level falls below the exit tube.

### III. COFFEE VOLUME AND TEMPERATURE HISTORY

#### A. Coffee extraction at constant heating power

We first assume that the temperature of the vessel increases linearly with time, as would be expected in an idealized situation where the pot is placed on a hotplate or stovetop at a given power setting, there are no heat losses, and the whole assembly is at a uniform temperature. This simplification is largely a matter of convenience, and a more realistic treatment of the heat losses and heat transfer could be incorporated into the numerical integrations, but no new insights are gained by doing so. Despite the idealization, measurements<sup>2</sup> show that a constant temperature rise rate (defined as  $\beta$ ) is a good approximation of typical practice. The standard value of  $10^\circ\text{C}/\text{min}$  was chosen for  $\beta$  for the calculations, which corresponds to the temperature rise rate with a medium power setting on the stove that was used. The effects of different heating rates is examined in the Appendix. For the coffeemaker shown in Fig. 1, the total volume available in the pressure vessel is 373 ml, the volume of the lower and upper connecting tubes is 20 ml, and  $\approx 28$  ml of water is needed to saturate the coffee plug. Thus, a total of 48 ml of water needs to be displaced before coffee will appear in the top collecting vessel. In the calculations the unfilled volume was varied from 30 ml to 225 ml and the coffee-plug conductance factor  $\alpha$  was taken to be  $1 \times 10^{-4} \text{ ml} (\text{Pa s})^{-1}$ , which we define as  $\alpha_s$  (see Sec. IV).

In the first calculation the vessel is sealed with water at room temperature ( $20^\circ\text{C}$ ). Figure 3 shows how  $P_{\text{tot}} - P_A$  depends not only on the temperature, but also on the volume of free space above the water. The pressure increases with temperature as expected, but for typical conditions, where  $V_i$  is of the order of 100 ml,  $P_{\text{tot}} - P_A$  is less than 20% of the increase in the saturated water vapor pressure in the range of interest, in contradiction to expectations that the overpressure in the vessel will always mirror the saturated vapor pressure versus temperature curve. For example, in going from  $20^\circ\text{C}$  to  $95^\circ\text{C}$ , the increase in saturated vapor pressure is 85 kPa, but the calculated overpressure is only 15 kPa.

Figure 4 shows how the volume of water displaced follows the pressure curves of Fig. 3. For an initial unfilled

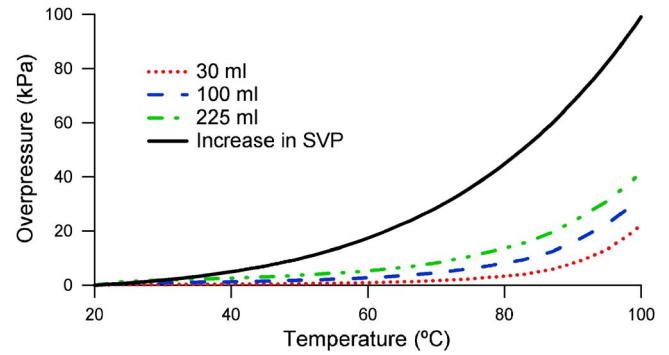


Fig. 3. The overpressure as a function of the water temperature and the initial unfilled volume. The increase in the saturated vapor pressure of water is shown for comparison. The large differences between this curve and the others is due to expansion effects of the air initially sealed into the vessel.

volume of 100 ml, no coffee appears in the top vessel until 48 ml of water has been displaced, at  $67^\circ\text{C}$ , or 4.7 min after heating commences at the standard rate of  $10^\circ\text{C}/\text{min}$ . Although the water vessel temperature is  $67^\circ\text{C}$  at this time, water begins to enter the coffee plug almost immediately after heating commences, albeit at very low flow rates. The best estimator of the relevant average extraction temperature from the calculations is the volume-weighted average extraction temperature of the first 100 ml of water into the plug, because this volume is what eventually emerges as enough for two shots of coffee. This volume-averaged temperature is just  $69^\circ\text{C}$ , substantially lower than the recommended range of  $90^\circ\text{C}$ – $95^\circ\text{C}$ , so it is unlikely that all the desirable flavors will have been extracted. We could go on extracting more coffee until the water is exhausted, but it is unlikely that the increasingly diluted result will be of high quality. (Note that the calculated extraction temperatures will be overestimated because the water first entering the plug will be cooled by the coffee grounds, which are unlikely to have had time to be warmed to the temperature of the vessel. Simple mass and specific heat considerations suggest that this effect could further lower extraction temperatures by  $5^\circ\text{C}$ – $10^\circ\text{C}$ , thus exacerbating the trends we have noted.)

Figure 4 also shows that if the vessel is underfilled with water, leaving 225 ml free space, then the 100 ml of coffee

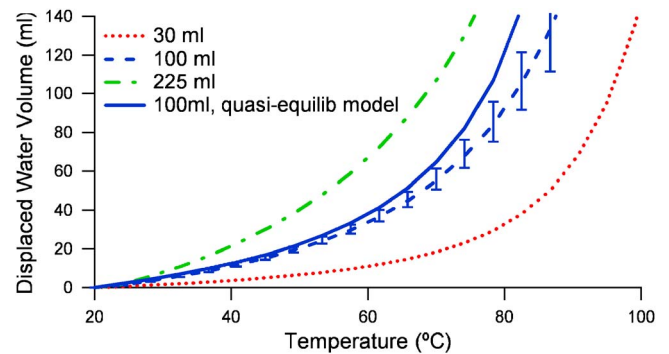


Fig. 4. The volume of the water displaced as a function of the water temperature for several values of the unfilled volume for an initial temperature of  $20^\circ\text{C}$ . The error bars for 100 ml free space show the variation in the calculated volumes if either the plug conductance  $\alpha$  or heating rate  $\beta$  is varied by a factor of 2 around the values used of  $1 \times 10^{-4} \text{ ml} (\text{Pa s})^{-1}$  and  $10^\circ\text{C}/\text{min}$ . The solid line is for  $V_i = 100$  ml calculated via the quasiequilibrium model described in Sec. III D.

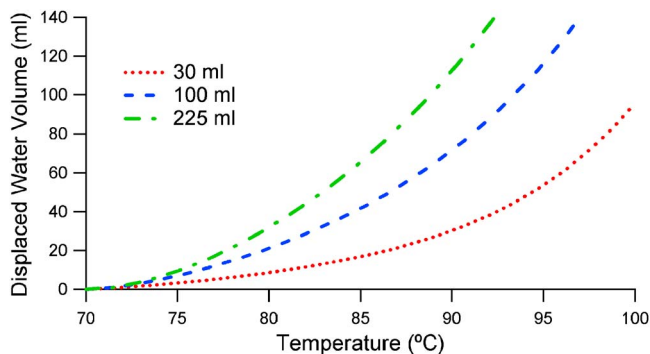


Fig. 5. The volume of coffee in the top vessel as a function of the water temperature for several values of the unfilled volume for an initial temperature of 70 °C.

will be extracted at temperatures of 12 °C lower, which is even less desirable. In the coffeemaker studied, 225 ml is about the maximum amount of airspace that can be left and still have sufficient water to produce 100 ml of coffee before the water level drops below the exit tube. These temperature and filling conditions are therefore just those required to make coffee where the traditional “gurgling” sound signals an automatic end to extraction.<sup>2</sup> It is unlikely to be fully flavored.

These calculations are applicable for a fixed value of the plug conductance  $\alpha$ , namely  $\alpha_i$ . The pressure at a given temperature will be higher or lower than shown in the three primary curves of Fig. 4 depending on whether the plug is packed more tightly (low  $\alpha$ ) or more loosely (high  $\alpha$ ). The pressure also depends on the temperature profile—the greater the temperature rise rate, the higher the pressure because the pressure rises faster than the exiting water flow rate can alleviate it. The Appendix shows how the ratio of the plug conductance to the temperature rise rate, together with the initial free space volume, determine the departure from the saturated vapor pressure curve. The error bars on the curve for 100 ml in Fig. 4 show how the calculated pressure rise is affected by assuming values for either  $\alpha$  or  $\beta$  which are a factor of two greater than or less than the standard values. The variations are not negligible, but they are not so large that they change the basic nature of the conclusions, namely that standard fill conditions result in extraction temperatures that are too low.

From the point of view of extracting coffee at higher temperatures, Fig. 4 shows that minimizing the free space is a useful strategy, although the volume-weighted average temperature is still only 81 °C for 30 ml of free space. In most models it is undesirable to fill to this level from a safety point of view because the water would cover the pressure-relief valve designed to vent steam if overheating occurs.

Increasing the temperature of the coffee extraction can also be achieved by warming the water above room temperature before sealing the vessel and heating further. Figure 5 shows how the water flow varies with temperature when the vessel is sealed with water at 70 °C. It can be seen that the coffee is extracted much closer to the ideal temperature. Although the volume-weighted mean extraction temperature is only 88 °C for a free space volume of 100 ml, the temperature of the water for the last few milliliters of extraction is calculated to be 94 °C. Thus these conditions are likely to be the best compromise between having the water so cold that

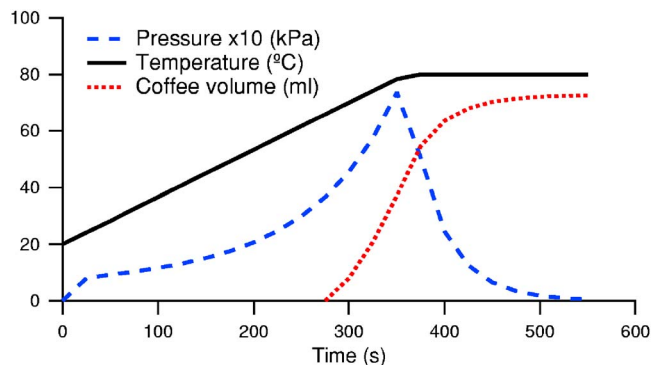


Fig. 6. The overpressure and coffee volume for the temperature profile shown;  $V_i=100$  ml. The volume of coffee was taken to be 48 ml less than the volume of the water displaced.

the flavors are not extracted at the beginning and so hot that a burnt flavor is risked toward the end of extraction.

Thus there are two methods of ensuring high coffee extraction temperatures: start with water at room temperature and leave little free space (not recommended for safety reasons), or preheat the water before sealing and leave modest free space volumes in the pressure vessel.

## B. Coffee extraction with an equilibrium final temperature

If the vessel is heated to a fixed temperature and thereafter maintained at that temperature, there is a point at which coffee ceases to be extracted even though there is water remaining in the pressure vessel. Figure 6 shows an example where the vessel has been heated from room temperature to 80 °C and maintained at that temperature. It can be seen that the pressure rises with temperature until the equilibrium temperature is reached, after which the pressure gradually drops back to zero as the continuing water displacement reduces it. The conditions for this equilibrium point, which provide a convenient experimental test of the analysis, can be examined by setting  $dV/dt=0$ . From Eq. (8), this equilibrium occurs when:

$$P_{\text{sat}}(T) + P_A \left[ \frac{V_i T}{V T_i} - 1 \right] - \frac{P_{\text{sat}}(T_i) V_i T}{V T_i} = 0. \quad (9)$$

We have

$$V = V_i + V_c + V_d, \quad (10)$$

where  $V_c$  is the volume of the coffee, and  $V_d$  is the volume of the dead space that has to be filled before any coffee appears in the top vessel. Equation (10) leads to a relation between the amount of coffee extracted and the final equilibrium temperature:

$$V_c = V_i \left[ \frac{T(P_A - P_{\text{sat}}(T_i))}{T_i(P_A - P_{\text{sat}}(T))} - 1 \right] - V_d. \quad (11)$$

The result in Eq. (11) is independent of  $\alpha$  and the temperature rise rate. This independence makes sense because both these variables determine how long it takes for the coffee to be extracted, but not how much. The singularity at  $P_{\text{sat}}(T) = P_A$  (that is, boiling) is expected because it is only for infinite free space volume that equilibrium can occur at this temperature because only then can air effects be ignored.

Note the singularity applies only to equilibrium conditions—there is no singularity in Eq. (8) as the temperature moves through the boiling point (which will be slightly elevated due to the increase in pressure in the vessel).

If the final equilibrium temperature is less than 100 °C, Eq. (11) also shows that the volume of water displaced, which is equal to the coffee volume seen in the top vessel plus the dead space, depends linearly on the unfilled volume when the vessel is sealed. This result gives rise to the interesting conclusion that for a given equilibrium temperature of the vessel, the higher the initial water level, the less the total coffee volume that can be extracted. Application of Eq. (11) shows that if the pressure vessel is filled so that only 5 ml is left unfilled, then even heating to 95 °C will not displace enough water to wet the coffee plug, and no coffee will be produced.

The application of Eq. (11) gives rise to a third way to optimize the coffee-making conditions. For example, if the water is preheated to 85 °C, a free space of 100 ml is left after filling, and the vessel heated to and maintained at 95 °C, then the amount of coffee produced will be about 115 ml. The difficulty with this method is that the increase in pressure toward equilibrium is very slow (see the following) so that it takes many minutes to extract the last few milliliters.

### C. Departure from equilibrium

Perturbation theory<sup>7</sup> can be used to find out how quickly the system departs from and approaches the equilibrium point described by Eq. (11). If

$$V(t) = V_e + v(t), \quad (12)$$

where  $V_e$  is the equilibrium solution to Eq. (11) and  $v(t)$  describes the small departure from equilibrium, we obtain by substituting Eq. (12) into Eq. (8) that

$$\frac{dv}{dt} + \frac{\alpha V_i T [P_A - P_{\text{sat}}(T_i)] v}{V_e^2 T_i} = 0. \quad (13)$$

It follows that the time constant  $\tau$  is given by

$$\tau = \frac{V_e^2 T_i}{\alpha V_i T [P_A - P_{\text{sat}}(T_i)]}. \quad (14)$$

We use Eq. (11) and make the approximation that  $P_A - P_{\text{sat}}(T_i) \gg P_A - P_{\text{sat}}(T)$  for most ranges of  $T$  and  $T_i$ , and obtain

$$\tau \approx \frac{V_i T [P_A - P_{\text{sat}}(T_i)]}{\alpha T_i [P_A - P_{\text{sat}}(T)]^2}. \quad (15)$$

Note that  $\tau$  depends inversely on  $\alpha$ . For typical conditions  $\tau$  is the order of 10 s initially, increases to the order of a minute (consistent with the curve shown in Fig. 6) by the time the temperature is around 80 °C, and again has a singularity at 100 °C for the reasons given previously.

### D. Quasiequilibrium conditions

An examination of Fig. 3 shows that for the typical situation when  $V_i = 100$  ml and  $T_i \sim 20$  °C, the overpressure is very much less than the increase in the saturated vapor pressure up to temperatures as high as 80 °C. This result implies that the large increase in the saturated vapor pressure over this range is effectively canceled by the almost equally large

decrease of the partial pressure of the air, leaving a relatively small positive pressure that forces the water through the plug at a rate fast enough to allow the air to expand and maintain the balance. Thus the system is effectively in quasiequilibrium even as the temperature changes, at least at the lower end of the temperature range, with the relation between the displaced volume and the temperature at any time approximately described by the equilibrium relation of Eq. (11). In terms of the pressures and volumes inside the vessel, the situation is equivalent to the one that would arise if there were no plug impedance at all (that is,  $\alpha = \infty$ ). In Fig. 4 the displaced water volume calculated from this quasiequilibrium model for  $V_i = 100$  ml is shown in comparison to the solution from the full dynamic model. As expected, at the lower temperatures it is in good agreement. Significant differences appear only at higher temperatures where the saturation vapor pressure rises faster than the compensating fall in the partial air pressure.

Given that the conventional mode of operation of the coffee-maker typically occurs in the temperature range up to 80 °C, the quasiequilibrium model provides a simple but useful description of the behavior under these conditions (particularly the influence of the initial air space volume on extraction temperatures). Its accuracy would diminish for more tightly packed coffee plugs and higher heating rates, and it is not accurate for describing the recommended practice of preheating the water to obtain higher coffee extraction temperatures.

## IV. EXPERIMENTS

Three sets of experiments were performed to test our analyses. In the first set, the flow conductance  $\alpha$  was measured. In the second set, measurements of both the time and the temperature at which coffee first appears were made. These measurements are used to test both the dynamic and quasiequilibrium analyses. In the third set of experiments, the basic equilibrium theory was examined in a series of measurements designed to test the predictions of Eq. (11).

The conductance  $\alpha$  was measured by removing the basket (which was 63 mm in diam. and 28 mm deep) and its saturated coffee plug after each extraction and then connecting it to a water tank with a measurable pressure head. The resultant flow was measured by weighing the water collected in a time interval of 1–3 min, depending on the flow rate. To keep the plug intact under pressure, it was either clamped against the seal of the coffee vessel, or inverted and placed on a wad of absorbent paper. Experimental variability arises from the difficulty of maintaining a tight pressure seal during the several minutes of the experiment, the inevitable variability in compacting the ground coffee in the plug for each run, and differences in the viscosity of water due to different temperatures, which varied from 18 °C–21 °C for the majority of the runs. Experiments showed that  $\alpha$  could vary by a factor of two greater than or less than the standard value depending on the coarseness of the grind and the amount of pressure used to tamp the ground coffee into the basket. For most of the data shown in Fig. 7, two packets of a commercial blend were used to limit the variability of the grind. Despite the potential for variability in these simple experiments, the data of Fig. 7 show that assuming a linear relation between the pressure difference and flow is not unreasonable for conditions when the plug is fully wetted. The value of  $\alpha$

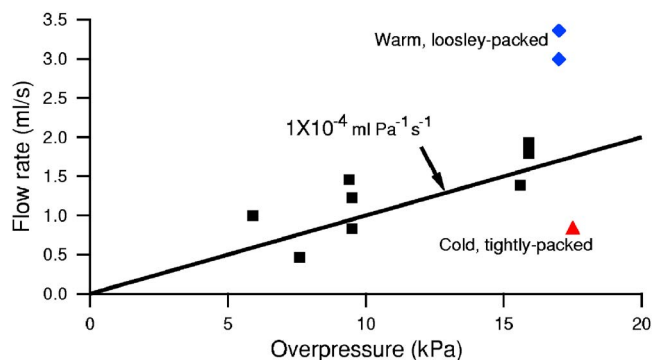


Fig. 7. Measurements to determine the flow conductance factor  $\alpha$ . The line shown (which is not the best fit line) is for the value of  $\alpha$  used in the calculations. The single triangular point was obtained after packing the basket as tightly as possible and water at 9 °C. The two diamond points above the line were obtained by performing the experiment on the plug immediately after coffee first appeared when the plug was still warm and less compacted.

used in the calculations,  $1 \times 10^{-4} \text{ ml (Pa s)}^{-1}$  is also reasonable, particularly because  $\alpha$  is not a sensitive variable for testing the analyses.

The triangular data point significantly below the line in Fig. 7 was obtained when packing the plug as tightly as possible, and with water forced through it at a temperature of 9 °C. Given that the viscosity of water is about three times higher at this temperature than at 60 °C, that the measurements were taken after the coffee had swollen and compacted, and that the coffee basket was packed as tightly as possible, the value of  $5 \times 10^{-5} \text{ ml (Pa s)}^{-1}$  implied for  $\alpha$  is probably as low as we could expect for a basket of the dimensions given, providing an extreme lower bound for practical calculations. The diamond-shaped data points above the line were obtained by stopping the extraction immediately after coffee first appeared on two occasions, and then performing the flow test while the basket was still warm and when the coffee was not completely swollen and compacted. The value for  $\alpha$  obtained for these measurements is about  $1.5 \times 10^{-4} \text{ ml (Pa s)}^{-1}$ , or 50% greater than the standard value used. Thus the likely range of  $\alpha$  under normal conditions for a basket of the dimensions noted is unlikely to differ by no more than a factor of two from  $1 \times 10^{-4} \text{ ml (Pa s)}^{-1}$ ; the impact of using values in this range on the standard calculations was shown in Fig. 4.

In the second set of experiments, the vessel was filled with varying amounts of water, generally at temperatures close to 15 °C, placed on the stovetop, and then allowed to warm until the first coffee emerged. At that time the time interval between the onset of heating and the first emergence of the coffee was noted, and the unit quickly disassembled and the temperature of the remaining water in the pressure vessel recorded. The water volume was varied from 150 to 350 ml, corresponding to initial free space volumes from 223 to 23 ml, respectively. The difference between the initial and final temperatures and the time taken were used to calculate the temperature rise rate, and the calculation was then run with these conditions and  $\alpha = 1.5 \times 10^{-4} \text{ ml (Pa s)}^{-1}$ .

The comparison between the calculated and measured times to the first appearance of coffee is shown in Fig. 8. The error bars indicate the sensitivity of the calculations to the factor of two variations in the value chosen for  $\alpha$  and a

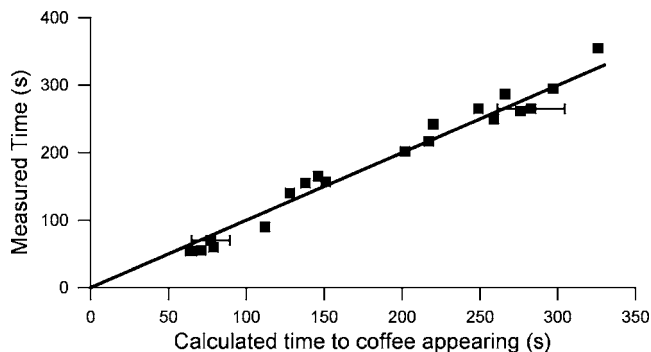


Fig. 8. Comparison of the calculated and measured times from the onset of heating to the first appearance of coffee. The free space volumes were varied from 23 to 223 ml. The data are around the theoretical line shown, and the error bars shown are typical for the data in the low and high time intervals. Uncertainties in the measured times are negligible.

temperature error of 3 °C. The error in measuring the time is negligible on the scale shown. The agreement is very good.

The calculated and measured temperatures of the water when the coffee first appears are shown in Fig. 9 for two sets of calculated times. The diamond points were calculated by integrating the dynamic equation [Eq. (8)] until 48 ml of water was displaced. In addition, the equilibrium description of Eq. (11) was solved for the temperature at which  $V_c = 0$  for each set of values of  $V_i$  and  $T_i$  to give the quasiequilibrium temperature at which coffee would be expected to first appear. These values are shown as triangular points in Fig. 9. Although the full dynamic solution comes closer to matching the measured temperatures, the quasiequilibrium calculations are surprisingly good, typically being 3 °C or 4 °C less than the temperatures calculated using the full dynamic model. Both sets of data tend to fall off at lower temperatures. Possible explanations for this decrease include the circumstances that produce coffee at cool temperatures have less water (more air space) initially and the system cools more quickly if there is little water remaining. Also when the coffeemaker is opened, excess water in the plug, which is cooler than that in the pressure vessel, flows back down before it can be removed, resulting in cooler measured temperatures.

Note that the highest water temperature measured when coffee first appeared was 80 °C (for the extreme case of an

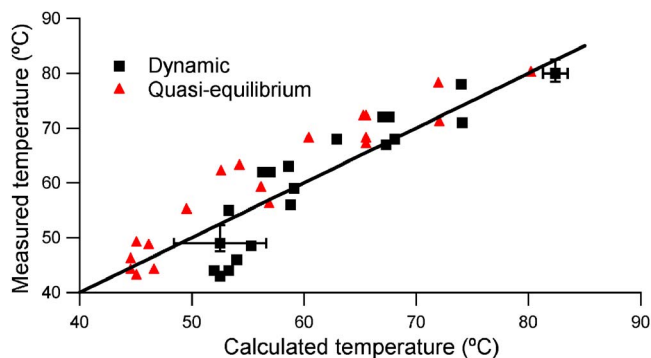


Fig. 9. Comparison of the measured temperature of the water in the pressure vessel when coffee first appears in the top container to the corresponding temperature calculated by two different methods. The solid line shown is the theoretical line expected, and the error bars shown are typical for the data in the low and high temperature ranges.

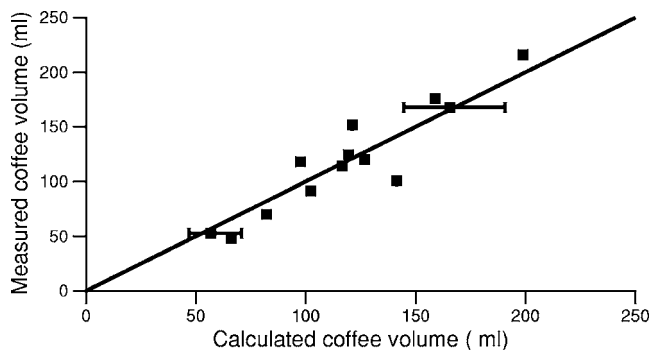


Fig. 10. Comparison of the measured and calculated coffee volumes for various equilibrium situations. The data fall around the theoretical line shown. The error bars are typical for the data in the low and high temperature ranges. The errors are largely due to the uncertainty of the temperature measurements.

initial free space of just 23 ml), consistent with the general thesis that extraction temperatures are in general too low if started with cooler water.

In the third set of experiments, the equilibrium relation suggested by Eq. (11) was tested using simple equipment available in most kitchens. The coffeepot was immersed in a large saucepan serving as a water bath at constant temperature. Temperatures were measured using a kitchen thermometer calibrated using an ice bath and boiling water. The total volume of coffee that appeared in the top vessel was measured after waiting for at least 10 min for equilibrium to be established, in line with the analysis of Sec. III C. Volumes were measured using a kitchen measuring flask with a graduated scale that was calibrated using a kitchen scale, estimated to be accurate to  $\approx 2\%$ . Standard laboratory equipment would improve accuracies considerably.

In these experiments the initial temperature of the water was varied between  $17^\circ\text{C}$  and  $75^\circ\text{C}$ , the final temperature between  $69^\circ\text{C}$  and  $96^\circ\text{C}$ , and the initial unfilled volume from 25 to 195 ml. A measurable volume of coffee was produced in each experiment, even when the final temperature was as low as  $69^\circ\text{C}$ , again contradicting the hypothesis<sup>2</sup> that the water temperature has to reach  $100^\circ\text{C}$  before coffee will emerge. The results are shown in Fig. 10, where the measured coffee volume is plotted against that expected from Eq. (11). There is good agreement between theory and experiment, with the data close to the theoretical curve. The error shown on the calculated volume arises mostly from the  $1^\circ\text{C}$  uncertainty in the temperature measurement.

## V. SUGGESTED EXTENSIONS

It would be a useful exercise for students to either perform similar experiments on other coffeemakers or to predict how they might work on the basis of a theoretical analysis. For example, a variant has a tap on the inner pipe beyond the coffee plug. If the tap is left open, the operation is identical to the model described earlier. If closed, pressures of several atmospheres are possible. Students could be asked to describe what happens when the tap is opened, to think of both the advantages and disadvantages of extracting coffee at higher pressures, and to suggest what would be the correct pressure at which to open the tap if  $95^\circ\text{C}$  was the desired temperature. Another model, the classic “Atomic,”<sup>8</sup> has a small hole, approximately 1 mm diam., in the inner tube in-

side the pressure vessel above the water level and below the coffee plug. Students could be asked about the effect that this hole would have on the pressure across the plug, its impact on the temperature of the first water to enter the coffee plug, the consequences of heating the vessel very slowly, and why the hole needs to be small. Experiments that determine the plug conductance factor  $\alpha$  indirectly via perturbation theory would also be easy to design, as would extensions to observe the effects of different coarseness of the grinds on  $\alpha$ . It would also be challenging to predict the differences in outcomes of a larger coffeepot based on changes in  $\alpha$ , free space volumes, and heating rates, and then to test their predictions experimentally using available models.

## VI. CONCLUSIONS

Simple application of the gas laws shows that some counterintuitive characteristics of a common and simple espresso machine can be explained. Specifically, our conclusions include the following: (1) The overpressure is almost universally less than the increase in the saturated vapor pressure for water at that temperature. It is difficult to envisage circumstances in which a value equal to the increase in the saturated vapor pressure could be attained. (2) For a given final temperature that is less than the boiling point and within the normal range of filling volumes imposed by the geometry of the vessel, less coffee can be extracted if more water is placed in the vessel before sealing. (3) Under conventional conditions of operation where the vessel is sealed with water at room temperature, and the vessel heated at about  $10^\circ\text{C}-20^\circ\text{C}/\text{min}$ , coffee will generally be extracted at temperatures of the order of  $60^\circ\text{C}-70^\circ\text{C}$ , too cold to extract all of the desired flavor. (4) For these same conditions, a quasiequilibrium model predicts displaced water volumes that are within 20% of those predicted from the full dynamic model. (5) Boiling the water is not a pre-requisite for producing coffee. (6) Variations in the degree of compactness of the coffee plug and the rate of heating will not change the amount of coffee extracted for any given equilibrium final temperature. They will affect the rate at which the coffee is produced, particularly at higher temperatures, but not to a significant degree.

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## APPENDIX A: CONDITIONS UNDER WHICH THE OVERPRESSURE MIGHT APPROACH THE INCREASES IN THE SATURATED VAPOR PRESSURE

If we start with the expression for the total pressure,

$$P_{\text{tot}} = P_{\text{sat}}(T) + \frac{[P_A - P_{\text{sat}}(T_i)]V_i T}{VT_i}, \quad (\text{A1})$$

and differentiate with respect to temperature, we have

$$\frac{dP_{\text{tot}}}{dT} = \frac{dP_{\text{sat}}}{dT} + \frac{[P_A - P_{\text{sat}}(T_i)]V_i}{VT_i} - \frac{[P_A - P_{\text{sat}}(T_i)]V_i T}{V^2 T_i} \frac{dV}{dT}. \quad (\text{A2})$$

We make the substitution for the overpressure,  $P_0$ ,

$$P_0 = P_{\text{tot}} - P_A, \quad (\text{A3})$$

and recast Eq. (1) as

$$\frac{dV}{dT} = \alpha P_0 / \beta, \quad (\text{A4})$$

and obtain

$$\frac{dP_0}{dT} = \frac{dP_{\text{sat}}}{dT} + \frac{[P_A - P_{\text{sat}}(T_i)]V_i}{VT_i} - \frac{\alpha[P_A - P_{\text{sat}}(T_i)]V_i T P_0}{\beta V^2 T_i}, \quad (\text{A5})$$

where  $\beta$  is the temperature rise rate. The two positive terms on the right-hand side are due to the rising vapor pressure and rising air temperature respectively; the negative term is due to the increasing volume into which the air can expand. Given that it is the second and third effects that are generally ignored, particularly the third, it is interesting to examine the conditions under which it is valid to do so.

Because  $P_{\text{sat}}(T_i) > 0$ , and  $V_i < V$ , we have

$$\frac{[P_A - P_{\text{sat}}(T_i)]V_i}{VT_i} < \frac{P_A}{T_i}, \quad (\text{A6})$$

and the right-hand side of Eq. (A6) has a value of 345 Pa/°C for  $T_i = 20$  °C. For temperatures less than 38 °C, this rate is greater than the slope of the saturated vapor pressure curve [first term of Eq. (A5)], so that at cooler temperatures, the heating of the air in the vessel contributes more to the pressure rise than the increase in the water vapor pressure. It is only at higher temperatures when both  $dP_{\text{sat}}/dT$  is larger and  $V_i \ll V$  that the second term contributes less, dropping to typically less than 15% by 90 °C.

The condition for the third term to be ignored is that

$$\frac{\alpha[P_A - P_{\text{sat}}(T_i)]V_i T P_0}{\beta V^2 T_i} \ll \frac{dP_{\text{sat}}}{dT}. \quad (\text{A7})$$

If the Clausius–Clapeyron description of  $P_{\text{sat}}$  is used, Eq. (A7) can be written as

$$\frac{\alpha[P_A - P_{\text{sat}}(T_i)]V_i T P_0}{\beta V^2 T_i} \ll \frac{LMP_{\text{sat}}}{RT^2} \quad (\text{A8})$$

or

$$\frac{\alpha R [P_A - P_{\text{sat}}(T_i)] V_i T^3 P_0}{\beta L M V^2 T_i P_{\text{sat}}(T)} \ll 0.1 \quad (\text{A9})$$

if the overpressure is to be within 10% of the change in the saturated vapor pressure. If we use the values for  $\alpha$  and  $\beta$

described elsewhere in this article, take  $T_i = 20$  °C, and the most favorable values for  $V_i$ ,  $V$ , and  $T$  to allow the condition to be met (30 ml, 140 ml, and 20 °C respectively), Eq. (A9) reduces to

$$\frac{P_0}{P_{\text{sat}}(T)} < 0.06. \quad (\text{A10})$$

Although this inequality can occur during the very early phases of brewing when  $P_0$  is small, the inequality breaks down typically within a few seconds. [We also note the logical inconsistency of having  $P_0 \approx P_{\text{sat}}(T)$  at high temperatures (the original hypothesis) and satisfying Eq. (A10)]. Therefore, the conditions under which the assumption of assuming the overpressure to be equal to the increase in the saturated vapor pressure of water require any of the variables or the appropriate combination of the variables in Eq. (A9) to differ by a factor of  $\approx 16$  or more if we want equality within 10%. Computation shows finding such a combination is difficult to do;  $\alpha$  can possibly be reduced by a factor of two by grinding the coffee very finely and compacting it as much as possible. It is possible to increase  $\beta$  by a factor of two or three using a standard stove, and the unfilled volume maximized at 225 ml. However, the calculations show that even for this combination of parameters, the overpressure is still 30% less than the increase in the saturated vapor pressure going from 20 °C to 95 °C.

We conclude that it is almost impossible to have the overpressure equal to the rise in the saturated vapor pressure, despite common expectations that it will do so. We also note that the calculation of the plug parameters in Ref. 2 is probably incorrect, because it assumes that the measured flow occurs under a pressure differential of one atmosphere, whereas it is likely that the pressure difference was considerably less.

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