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Oscillating droplets and incompressible liquids: slow-motion visualization of experiments with fluids

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Abstract
We present fascinating simple demonstration experiments recorded with high-speed cameras in the field of fluid dynamics. Examples include oscillations of falling droplets, effects happening upon impact of a liquid droplet into a liquid, the disintegration of extremely large droplets in free fall and the consequences of incompressibility.

Online supplementary data available from stacks.iop.org/PhysED/47/664/mmedia

Introduction
The recent introduction of inexpensive high-speed cameras [1] offers a new experimental approach to many simple but fast-occurring events in physics (e.g. [2–6]). As an extension of experiments in the field of mechanics of solids [3], we present examples with fluids. The primary goals are first to make the method well known in physics teaching and second to make teaching materials available to those who have no access to such cameras by providing multimedia supplements on the web (available at stacks.iop.org/PhysED/47/664/mmedia). In addition we want to emphasize that just showing such experiments is not enough, rather the underlying physics must be discussed in as much detail as possible in order to provide background information for the teachers. The results of such experiments should offer more than just nice images or videos.

Oscillations of liquid droplets
In 1954, McDonald [7] argued that one of the few predictions a meteorologist can make with 100% accuracy is the following: ‘...ask an illustrator or cartoonist to draw a falling raindrop. His picture will be dead wrong’. This statement is still supported now by many illustrations of falling raindrops, which depict them as having an aerodynamic streamline shape, although the correct shape has been known now for more than 50 years [7]. In brief, droplets with diameters below 1 mm are more or less spherical. But larger droplets have more oblate shapes and are furthermore oscillating when achieving their terminal velocities [8–11]. High-speed cameras were used to overcome the wrong preconception and they are also the ideal tools to investigate the reality in simple experiments. Surprisingly, experiments on shape oscillations of raindrops
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Figure 1. Droplet of red wine, created at the end of a funnel and observed with a high-speed camera at 4000 fps. Top row from left to right: liquid column; creation of a droplet; 16 ms later, a large and above a smaller droplet are formed. Bottom row: +16 ms (left) and +42 ms (middle) later as well as the same scene with a ruler.

are still studied [10]. The reason behind this is that shape changes have practical consequences in meteorology: modern methods to determine the amount of precipitation using radar need to know the static equilibrium form of droplets as the input parameter. Therefore precise knowledge of the dynamic behaviour of falling raindrops is needed.

When raindrops fall to the ground, they can reach terminal velocities—depending on their size—which can amount to about 10 m s$^{-1}$. For these cases, a detailed investigation of the shape changes with high-speed cameras requires vertical wind tunnels [10], since otherwise the droplet will fall through the field of view of the camera too quickly to observe even a single oscillation. For teaching purposes one may, however, use the initial rather than the final phase of droplet fall. Droplets are formed initially with a nonspherical shape from a faucet or a similar device. They are accelerated by gravity and start to oscillate, which can be observed easily at these low speeds (e.g. [5, 7, 10]).

In our experiments an artificial faucet is used made of the lower end of a funnel covered by the skin of a balloon that has a tiny hole in it. The funnel above is filled to a height of 10–15 cm, which provides enough pressure to produce droplets every 20 s or so (depending on the width of the hole). Sometimes a small capillary was also inserted into the balloon skin. Figure 1 (movie 1, available at stacks.iop.org/PhysED/47/664/mmedia) depicts some snapshots of a droplet being formed, separating from the faucet, and falling down. Initially the liquid forms a cylinder-like column until the interplay between gravity and surface tension leads to a narrowing of the cylinder and finally a tearing off of—in our case—a large droplet of average
A droplet of around 5 mm diameter oscillates between one form resembling a cigar and another more like a pancake (falling distance after creation around 5 cm).

The oscillation frequency depends on size: the small droplet (≈2 mm) undergoes a full period \( T \approx 12 \text{ ms}, f \approx 83 \text{ Hz} \), while the larger droplet (≈5 mm) has not changed that much in shape \( T \approx 33 \text{ ms}, f \approx 30 \text{ Hz} \).

diameter between 4 and 5 mm. A short time later, the residual part of the thread-like thin column on top of the drop produces a second much smaller droplet of around 2 mm diameter, sometimes a series of one or two more even smaller droplets of around 1 mm follow. These droplets are spatially all very well separated from each other, which allows us to study the dynamics of droplets of different sizes within the same set up. Positioning a ruler at the same distance as the droplets in the image allows us to perform rough quantitative estimates of droplet size. We recorded video sequences with 4000 frames s\(^{-1}\) and an integration time of \( 1/4000 \text{ s} \) (if integration time is \( 1/\text{framerate} \), one refers to the shutter as open). Since pure water caused contrast problems in the images, we used a deep red colour wine as the liquid (any coloured water would do). Figure 1 clearly shows a shape oscillation between prolate and oblate shape of the falling droplets.

For any measurement a compromise has to be found between falling distance and droplet size. On the one hand, image size should be large in order to observe several oscillation periods for higher accuracy. On the other hand it should be small such that the droplet size can be accurately guessed. Figure 2 (movie 2, available at stacks.iop.org/PhysED/47/664/mmedia) shows two snapshots of an enlarged section recorded for a different droplet (diameter ≈5 mm). One can clearly see the drastic shape variation of the droplet, but also that the form can only be very roughly approximated by a spheroid.

Figure 3 (movie 2, available at stacks.iop.org/PhysED/47/664/mmedia) depicts the same large droplet at a later time. Meanwhile, the second smaller droplet (diameter ≈2 mm) has entered...
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the image. Obviously it oscillates at a larger frequency than the 5 mm droplet.

From figures 1–3 it is obvious that oscillation frequencies are easy to measure. They do not depend on falling distance, we recorded similar frequencies for falling heights of 20 cm (figure 4) and of up to 160 cm. The respective video (movie 3, available at stacks.iop.org/PhysED/47/664/mmedia) also demonstrates that the smallest produced droplets of around 1 mm diameter do not oscillate as much but have more or less spherical shape.

Qualitatively, droplet oscillations have just recently been demonstrated for teaching (e.g. [5]), however, as with any such high-speed demonstration, it is important to not only observe but also understand and explain the phenomenon. Therefore, some background information on the underlying physics is presented.

The oscillation frequency \( f \) of droplets whose equilibrium shape is spherical dates back to Lord Rayleigh [12]:

\[
f = \sqrt{\frac{2\sigma}{\pi^2 \rho R^3}}.
\]

Here \( \sigma \) denotes surface tension and \( \rho \) the density of the liquid. \( R \) is the equivalent radius of the droplet, i.e. the one if it were a sphere. Lord Rayleigh derived this equation by calculating the effect of shape changes upon the constraint of mass and also volume conservation. Upon oscillating, the liquid surface area of the droplet increases, giving rise to potential energy related to surface tension. The movement of the liquid is furthermore related to a kinetic energy, which allows us to define the Lagrange function of the problem. Solving the Lagrange differential equation gives the dynamical behaviour and hence also equation (1).

The major features of equation (1) can, however, already be understood from a much simpler model which tries to model the droplet oscillation by a mass–spring system, i.e. a harmonic oscillator. The potential energy is related to a restoring force, whose ‘spring constant’ \( k \) is proportional to surface tension: \( k \propto \sigma \) when applying Newton’s law and one needs to know the moving mass. It makes sense that it is proportional to the total mass of the droplet: \( m \propto m_{\text{droplet}} = \rho R^3 \). Consequently, solving Newton’s law for this hypothetical spring mass system yields

\[
f \propto \omega = \sqrt{\frac{k}{m}}, \quad \text{i.e.} \quad f \propto \sqrt{\frac{\sigma}{\rho R^3}}.
\]

The prefactor depends on the geometry of the droplet and whether the fundamental or higher order harmonics are present. Since larger droplets do not resemble a sphere as the equilibrium form, equation (1) is only an approximation, which needs corrections. However, experiments with high-speed cameras have demonstrated that deviations from equation (1) are rather small [10]. Figure 5 depicts the theoretical expectation for the fundamental vibrational mode of water droplets with spherical equilibrium shape according to equation (1) using \( \sigma(20 \, ^\circ C) = 72.7 \, \text{mN m}^{-1} \) and \( \rho = 10^3 \, \text{kg m}^{-3} \).

Our experiments yielded values of about 83 Hz for a 2 mm droplet and 30 Hz for a 5 mm droplet, which roughly coincides with.
expectations. Probable reasons for deviations are as follows.

(1) The estimate of time for one period will have an error of <5% if more than one period is used.

(2) For the estimate of equivalent droplet size by comparison with the ruler, an error of 10% is possible since these droplets initially never have genuinely spherical shape. This leads to frequency errors of around 15%.

(3) Ethanol has a surface tension which is about a factor of three smaller than that of pure water (22.3 mN m\(^{-1}\)), therefore red wine with 12.5 vol% alcohol as a mixture of ethanol with water should have a somewhat lower surface tension, reducing the expected frequencies by about 5%.

(4) Finally, as mentioned above, the Rayleigh formula is only an approximation which does not apply perfectly [10].

Keeping these sources of error in mind the agreement of theory with experiment is quite satisfactory. Knowing that shape oscillations of droplets do also occur for raindrops near their terminal velocity, we suggest that next time when wandering in the rain and singing B J Thomas’s 1969 song *Raindrops Keep Fallin’ on My Head* you should think about the intricate physics behind the simple phenomenon of oscillating raindrops.

We also mention that very similar oscillation phenomena do occur for the inverse situation, i.e. gas bubbles within liquids rather than liquid droplets within gases. Oscillation of spherical air bubbles within water do furthermore lead to the characteristic sound waves associated with such turbulent liquids [13].

**Impact of a droplet onto a liquid**

Nearly every morning, an interesting physics phenomenon happens on thousands of breakfast tables around the world, mostly not admired but rather cursed. One wants to add some milk to a full cup of coffee and pours it in from a milk container held at some height above the cup in a hurry. And very often, Murphy’s law applies: it happens that a large drop of milk–coffee mixture is ejected from the liquid and ends up on the initially clean table cloth. In order to investigate this phenomenon in detail we reduce the poured liquid to a single droplet and let it fall into a glass filled with the same liquid. For contrast reasons we again choose a deep red wine.

Figure 6 (movie 4, available at stacks.iop.org/PhysED/47/664/mmedia (note this older video was recorded with ac current light bulbs which results in 100 Hz brightness variation) depicts some snapshots of the impact of a 4 mm falling (and oscillating) droplet on a liquid surface. The droplet started at a height of about 56 cm above the liquid surface and was created with the same mechanism as described in the section on oscillations of liquid droplets.

The first two images again clearly reveal the oscillation in shape. Shortly after impact (+6.5 ms) one can observe ejection of a so-called crown (see below) with a spray of small droplets being formed. This goes along with an indentation of the surface of the liquid (nicely seen e.g. at +27.5 ms). Afterwards, a vertical column of liquid is ejected upwards to a height of several cm. Upon falling back, a single droplet is formed which falls back a bit later. This also goes along with surface waves on the liquid. In order to better understand the physics after impact, figure 7 show the result of a similar experiment at the moment of maximum indentation of the liquid’s surface (contrast and colour enhanced with Adobe Photoshop). Clearly, the surface minimum height has shifted by nearly 15 mm. In the case of a smaller falling height of about 50 cm of
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Figure 6. A 4 mm droplet falls onto the surface of a still liquid. This results in a so-called ‘crown with spray’ deformation of the inner liquid’s surface and subsequently the ejection of a column which finally forms a single drop above the liquid. Recorded with 2000 fps, shutter open (see text for details).

Figure 7. A similar droplet impact from a height of 80 cm onto a liquid surface observed with 4000 fps, shutter 1/5000 s. The moment of maximum indentation is shown as a colour and contrast enhanced image. One can nicely see the crown and spray of the small droplets.

figure 6, the indentation was estimated to be around 11 mm.

The process with ejection of a column and droplets can be understood semiquantitatively (i.e. within errors of ±25% or so), as will be demonstrated for the experiment of figure 6. The length scale in the experiment is known from objects of a given size or where a ruler has been placed nearby, as in figure 7.

(1) The velocity of the droplets can be measured from the high-speed recording. In the example, we find directly above the surface $v \approx 3.3 \text{ m s}^{-1}$ (corresponding to a height of 56 cm).

(2) The kinetic energy of the droplet is estimated—from its diameter 4 mm and density $10^3 \text{ kg m}^{-3}$), giving a mass of 33.5 mg—to be $E_{\text{kin}} = \frac{1}{2}mv^2 \approx 1.8 \times 10^{-4} \text{ J}$.

(3) This kinetic energy upon impact is divided into various components. The first part is transformed into kinetic energy of the rising wall of the crown and the small spray droplets, another part into surface wave energy. Another small part may lead to thermal energy, but a large part is transferred into potential energy of the liquid. Upon impact, an inner surface is created, i.e. there is a change of surface area and hence—due to surface tension—a change of the potential energy of the liquid. Figure 8 gives a scheme of how the additionally created surface area can be estimated. The realistic form of the surface (black line) is approximated by a segment of a sphere. Input parameters are the height $h$ and the diameter $2a$ of the
indentation. The surface area of the segment of the sphere is given by $S = \pi (h^2 + a^2)$, i.e. the change with respect to a planar liquid surface is $\Delta S = S - \pi a^2 = \pi h^2$. From the measured value of $h = 10.8$ mm we estimate the maximum of the additional surface area, created upon impact to be $\Delta S \approx 366 \text{ mm}^2$. Knowing the surface tension of water to be $72.7 \text{ mN m}^{-1}$ at room temperature, we can estimate the potential energy due to surface tension and area increase to be $\Delta E_{\text{pot}} = \sigma \Delta S \approx 2.7 \times 10^{-5} \text{ J}$, which is about 14.4% of the available kinetic energy.

(4) The liquid surface now acts like an elastic membrane, i.e. like a spring: the increase in potential energy creates a restoring force, i.e. the surface swings back through its planar equilibrium position. Thereby a liquid column can be ejected. We can estimate the potential energy at maximum height of the liquid column. Assuming a measured column height of about 21 mm (i.e. centre of mass height $h_{\text{CM}}$ being half of that) and average column width of 4 mm, we find a column mass $m_{\text{col}}$ of about 260 mg, giving a potential energy of the column at its maximum height of $E_{\text{pot}} = m_{\text{col}} g h_{\text{CM}} \approx 2.61 \times 10^{-5} \text{ J}$, which is in agreement with the potential energy of the liquid surface. This means that the liquid potential energy is more or less equal to the ejected column potential energy.

We mention again that the estimates presented here are very approximate, they should only give an idea of the underlying physics. For example, the length measurements within the images give rise to errors and the column and indentation were approximated by the cylinder and sphere segment. We also used data for water, although red wine is a mixture of water with alcohol and has a different density and surface tension (see the section on oscillations of liquid droplets) etc.

Nevertheless, this description offers an explanation for the observed behaviour: while hitting the surface, part of the kinetic energy of a droplet is transferred into potential energy of the liquid surface. Due to surface tension it acts like a spring and ejects a water column. If the water in the column is too large for a single droplet to form, the column breaks up and occasionally a single drop forms on top of the column. Applying this to the milk–coffee problem: if the milk is poured not vertically but at an angle, it may happen that the droplets are ejected sideways with the respective consequences.

We also studied whether the liquid in the column or at the top of the column forming the separated droplet is mostly due to the incident droplet. For this purpose we let a red wine droplet fall into a beaker with pure water. The ejected column was already mixed, i.e. consisted of a strongly diluted wine.

Finally, we mention that the general topic of drop impact on surfaces has a variety of applications. For solid surfaces these include spray cooling, ink-jet printing, soil erosion due to rain, or high-speed impact erosion in steam turbines. Concerning liquid surfaces, research deals e.g. with rain formation and the interaction of rain with the surface of the ocean. An excellent review [14] discusses all aspects of drop impact for liquids.

In the scientific literature, the process discussed above is described as follows: for high enough impact energy of a drop on a liquid surface a crater is formed in the liquid. At the crater wall an unstable sheet of liquid is raised above the surface, which disintegrates into tiny droplets at the upper rim, a phenomenon called crown formation. A hemispherical cavity with a radius that can be much greater than the drop radius is formed and a liquid column rises out of the centre of the crater. The maximum height to which the central jet rises is a function of the...
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Figure 9. An exploding balloon filled with water and recorded with 2000 fps. The rupture of the skin proceeds very quickly within a few ms. The free fall of the ‘big’ water droplet follows the law of gravitation and is much slower.

impact energy and whether a drop can separate from its tip depends on the so-called Weber number \( We = \rho v^2 2R/\sigma \), where \( \rho \) is the density of the liquid, \( v \) and \( 2R \) the droplet impact velocity and diameter and \( \sigma \) the surface tension. If \( We \) is above a critical number, cited to be around 100, droplets may separate. In our experiment, the Weber number was around 500, which explains why we observed the drop separation.

Disintegration of very large droplets

Having seen many raindrops in nature and other liquid droplets in the laboratory, the natural question to ask is about the available size range. Water droplets in fog and clouds have typical sizes in the 10 \( \mu \text{m} \) range. This is not sufficiently large for gravity to dominate their behaviour. Raindrops falling from clouds usually have sizes of the order of 1–3 mm, in extreme cases 5 mm droplets may be observed. What about larger droplets? We performed an experiment which created an extremely large droplet by filling a regular balloon with about 2 l of water (horizontal diameter around 15 cm). In order to rupture the rubber skin, we could not use a green laser and a red balloon as is possible for an air-filled balloon \[3\] since the water has a large heat conductivity and efficiently cools the skin. Therefore a sharp needle was used. Figure 9 (movie 5, available at stacks.iop.org/PhysED/47/664/mmedia) shows a sequence of snapshots: after the needle has created a hole in the skin, a rupture evolves with the speed of sound in the material, here several 100 m s\(^{-1}\) \[3\]. The rubber skin has gone after a few ms, but the water inside, which was initially at rest, is now in free fall, which only covers about 8 mm in the first 40 ms after the start, giving a speed of only 0.4 m s\(^{-1}\). This large velocity difference is responsible for the strange sight in figure 9, which resembles a large liquid droplet, seemingly avoiding gravitation and hanging still in the air. Similarly, figure 10 from a different balloon shows that proper illumination may give rise to spectacular views of the water.

There are a few important details. First, when the balloon skin recedes, a spray of tiny droplets forms, which then surrounds the balloon-shaped large droplet. This is due to the adhesion forces between the rubber skin and the water: upon receding, the skin can accelerate adjacent parts of the water. We tried in vain to get rid of this effect. One possibility that failed was to cover the inner surface of the balloon with some hydrophobic liquid like oil. Unfortunately, the rubber material...
changes its properties upon such treatment. A very funny outcome of one respective experiment was that the needle made a hole in the balloon, but no fissure was created. Rather the water was just streaming out of the hole due to the hydrostatic pressure inside—and, of course, Murphy’s law was valid: the water hit the trousers of the physicist.

Second, the method of rupturing using a needle creates an artefact: the person with the needle can never be fast enough to remove the needle and hand, therefore they are seen in the whole sequence.

Third, at the beginning of the fall, the droplet did indeed have a shape, resembling what artists formerly thought was the shape of a falling droplet; but this only lasted a short while and was due to the fact that this particular droplet shape was created intentionally.

The big droplet had fallen out of the field of view of the camera before one could verify whether it stayed intact. We therefore repeated this experiment by letting the droplet fall from the second floor of our university building. Figure 11 shows two snapshots of the result. The 10–15 cm droplet stayed more or less intact for a falling distance of 5–7 m before disintegrating into many smaller droplets.

Why does a large droplet disintegrate at all? In brief, it depends on the interplay between gravitation, frictional forces during the fall and surface tension. It is well known that the largest observed droplets in rain showers are around 5 mm in diameter and the largest ones in the laboratory around 9 mm. The break-up mechanisms are, however, different for the two cases: in natural rain showers there is collision-induced disintegration and break-up [15,
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Figure 12. Scheme of the break-up mechanism for large droplets.

16], whereas single large droplets studied in the laboratory disintegrate due to an aerodynamic break-up mechanism [17, 18]. Qualitatively, it may be understood in the following way (figure 12): during their fall large droplets flatten such that a concave form develops at the bottom. In addition, shape oscillations are usually induced due to the applied forces. For a critical size (e.g. 9–10 mm) a large drop may then break-up into two smaller droplets of similar size. It is worthwhile mentioning that this droplet break-up scheme is very similar to the fission of large nuclei treated with the liquid drop model for atomic nuclei.

For our large droplets of initial volume of the order of 1 l, one can easily estimate that around 1 million droplets of about 1–2 mm diameter are formed (more or less, depending on the developing size distribution).

Incompressibility of liquids: shooting onto empty and liquid-filled objects

A well-known experiment of fluid dynamics visualizes the property of incompressibility of liquids by shooting, for example, into a raw egg which immediately leads to an explosion of the egg and spilling of its interior all over the place. Figure 13 (movie 6, available at stacks.iop.org/PhysED/47/664/mmedia) depicts an example recorded with a high-speed camera. The projectile from an air pistol, travelling at a speed of around 100 m s\(^{-1}\) hits the egg shell. Even before it exits again at the other side of the egg, the shell cracks and the liquid parts of the egg are ejected. Images of this kind are usually described qualitatively by stating that due to the incompressibility of the liquid, the induced interior pressure cracks the shell. Sometimes reference is made to Pascal’s law while only briefly mentioning that in reality the compressibility and shock wave travelling at the speed of sound must be applied [5]. The egg explosion experiment suffers from the drawback of many easy-to-perform hands-on experiments: they are extremely easy to perform, but the underlying physics can be quite complex. Still, the teacher presenting such experiments must know about the underlying physics.

In order to have reproducible conditions, we used Christmas-tree ornaments, i.e. spherical plastic cups of typically 57 mm diameter (inner diameter 55 mm), decorated with angels etc. We

Figure 13. Projectile from an air pistol hitting a raw egg. Top row: left start, then: +0.333 ms, bottom row: +1 ms, +1.333 ms (projectile visible), 6000 fps, shutter open.
first used air-filled samples. Figure 14 shows three snapshots of a sequence (movie 7, available at stacks.iop.org/PhysED/47/664/mmedia) recorded with 8000 fps in order to be able to estimate speeds of the projectile (mass $= 0.54 \text{ g}$, length $= 6.7 \text{ mm}$, length/diameter $\approx 1.5$, volume $= 6 \times 10^{-2} \text{ cm}^3$). The length scale was given by the size of the sphere outer diameter, 57 mm, as well as the length of the aiming device on top of the muzzle’s end (length 30 mm). The opening of the sphere is at the bottom.

Before hitting the sphere, the projectile speed was estimated to be around $v = 84 \text{ m s}^{-1}$. Due to the impact and rupture of the plastic sphere material it lost about 1/7 of its kinetic energy and the speed after exiting the sphere was only around $78 \text{ m s}^{-1}$. Within the 1/8000 s integration time, the projectile moved around 10.5 mm, which explains the observed projectile geometry, which has a ratio of length/diameter $\approx 3–3.5$, rather than 1.5 of the projectile at rest (however, since the whole sphere plus surrounding was imaged, we had only limited spatial resolution and this measurement should be considered qualitative only).

Since the interior of the air-filled sphere does not offer much resistance to the projectile, its main effect is to break holes in the plastic shell. The transit time through the sphere was estimated to be around 0.75 ms. Depending on the material, these holes can be quite small or—if cracks are formed—larger fragments with cm$^2$ areas may break off, such as is shown in figure 14. However, the main body of the sphere usually stays intact, i.e. the projectile only induced breaking upon impact.

Figure 15 (movie 8, available at stacks.iop.org/PhysED/47/664/mmedia) shows the same setup, the difference being that the sphere was filled with water and closed with a rubber stopper (opening at top). Let us first discuss the projectile: before hitting the sphere its speed was measured to be around 80–84 m s$^{-1}$ (the inaccuracy mainly due to the length measurements). After exiting the water-filled sphere, it was much slower and only propagated with around 47 m s$^{-1}$, i.e. the 55 mm of water led to a quite strong frictional force, slowing down the projectile (we will come back to this later). This can also be seen in the longer transit time of about 1 ms with regard to the 0.75 ms of the air-filled sphere. The main feature, however, is the breaking up of the sphere (figure 15(b)) even before the projectile exits the sphere again. This feature is important: the ‘explosion’ is initiated while the projectile is still within the sphere.

For the air-filled sphere cracks did not cover the surface, therefore it cannot be that the cracks formed when the projectile (travelling at the speed of sound within the outer shell) entered...
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the water-filled sphere, and the projectile cannot therefore be responsible. Rather the water must be the cause. The typical explanation of this behaviour requires the concept of compressibility $\kappa$ of the water inside the sphere. It describes the ability of the liquid to be compressed, written as volume change $\Delta V$ with respect to original volume $V$ due to an outer pressure difference $\Delta p$.

$$\frac{\Delta V}{V} = -\kappa \Delta p.$$  \hspace{1cm} (3)

The compressibility of water at room temperature amounts to about $0.5 \text{ GPa}^{-1}$. The projectile had a volume of about $6 \times 10^{-2} \text{ cm}^3$ (measured by adding 50 projectiles into a narrow water-filled glass cylinder with a volume scale), while the volume of the water within the completely filled sphere was about $87 \text{ cm}^3$. From the high-speed sequence, it is clear that only tiny amounts of water spill out within the first fraction of a ms after the projectile had entered. This means that water plus projectile must fit in the same volume as before. This is only possible if the water is compressed by at least the volume of the projectile (the compressibility of the projectile solid material is at least a factor of 20 smaller than that of the liquid, therefore we neglect volume changes of the projectile). It order to achieve this compression of the water, a pressure of about 140 atmospheres would be required. Obviously, the plastic shell cannot provide this pressure, i.e. it will crack.

The question of how long it will take to crack the shell depends on the speed at which the pressure shock waves travel within water. It will definitely have a minimum value given by the speed of sound in water, which is above $1480 \text{ m s}^{-1}$ at room temperature. Therefore the diameter of the sphere (57 mm) is traversed at least within $38.5 \text{ ms}$, which is well below the time resolution of our recording. This short time qualitatively explains why the cracks occur while the projectile is still within the sphere.

Unfortunately, this explanation is not totally satisfying—it still misses a critical point. Eggs usually have an air bubble within the shell and similarly, the sphere probably also contained an air bubble, which could easily be much larger than the volume of the projectile. Therefore there would be no need to compress the liquid, it could just redistribute itself within the given volume without exerting pressure on the shell. The first guess to deal with this problem is obvious: the water suffers from inertia, i.e. the timescale for moving the liquid within the shell is too long, i.e. the pressure waves reach the boundary way before a redistribution of liquid can even start.

This was tested with three simple experiments.
Figure 16. A projectile enters a water-filled sphere from the top (a). It starts to crack (b) before the projectile exits (c). Shortly afterwards, the sphere is completely destroyed (d) (see text for details).

(1) The Christmas-tree ornament experiment was repeated with a water-filled sphere, however this time the opening was not closed and the pistol was shooting vertically down. As the sphere was left open, water should in principle have enough room to redistribute by pushing water out of the opening. Also, by shooting into the opening the entry of the projectile did not cause any direct impact-related mechanical stress within the shell. One could observe (figure 16; movie 9, available at stacks.iop.org/PhysED/47/664/mmedia) that the shell had already cracked before the projectile exited at the bottom of the sphere, i.e. the sphere’s explosion must have been due to a pressure effect caused by the water alone.

Figure 16 also demonstrates that only a little bit of water exited through the opening after the projectile entered, i.e. the water spray volume was small. Unfortunately, a side effect disturbed the scene: when pulling the trigger of the air pistol, the projectile pushes the air out of the muzzle. This can be clearly seen in the video: an air stream is pushed out of the muzzle in front of the projectile about 2 ms before the projectile leaves the muzzle, and this air stream already causes some turbulence and waves on the water surface within the opening, causing a little bit of spray.

(2) A small 8 mm diameter steel sphere was dropped from a height of about 80 cm into a beaker filled with water. Figure 17 (movie 10, available at stacks.iop.org/PhysED/47/664/mmedia) depicts some snapshots. The sphere with a speed of about 3 m s\(^{-1}\) (falling height \(\approx\) 50 cm) upon entering the water was fast enough to create a tunnel, which was filled with air behind it within the water. This means that due to inertia, the water could not immediately close the gap behind the sphere whereas the air molecules could do so. The timescale for closing the gap was about 30 ms for this experiment. Afterwards, the caught air bubbles in the liquid took time (in our case 200–300 ms, depending on the height of the beaker) before they rose back to the surface of the water.
Figure 17. A steel sphere falling from a height of 50 cm into a beaker filled with water. An air tunnel is created behind the sphere (see text for details).

Figure 18. A projectile from an air pistol entering a 260 mm long filled water container: (a) just entering; (b) an air tunnel is formed behind the projectile; (c) the air tunnel behind the projectile starts to collapse; (d) the projectile hits the rubber skin (for more details, see text).

Of course the projectile of an air pistol is faster. Figure 18 (movie 11, available at stacks.iop.org/PhysED/47/664/mmedia) depicts the results of a third experiment with some snapshots of an air pistol shooting vertically down into a container of water of length 260 mm which is closed at the bottom with the skin of a rubber balloon. One clearly sees the same as with the falling metal sphere from figure 17: the projectile creates an air-filled tunnel behind itself and the adjacent water needs at least 5 ms to close it. At the end, best seen in the video, the air from the tunnel creates many air bubbles slowly rising to the surface.

There are of course also several differences in the sphere experiment: first, the open surface at the top is quite large (inner diameter around 30 mm), which allows the adjacent water to be pushed upward by the projectile, i.e. there will be a lot of spray coming off the top. Second, the
side walls were much thicker and the bottom was made from elastic rubber such that no explosion should take place. The rubber clearly showed some outward bulging by more than 2.5 mm vertically before the projectile hit the rubber, clearly indicating that the inner volume had to expand. If the bottom had been a thin solid material, it would have cracked in a similar way to the tree ornament. An artefact was also that a little bit of water was pressed out of the interface between the rubber and the lower end of the beaker due to the large internal pressure. These water droplets nicely scatter the light, which was incident from the side (see also video).

This experiment proves that the water does indeed suffer from inertia as anticipated. However, it also demonstrates that the projectile does not only reduce the available volume by its own volume but in addition by the water-free volume behind the projectile, which can be much larger than the projectile volume alone. Therefore, the pressure needed would increase even further, and the explosion of a solid shell is even more likely.

(For very fast projectiles, but not yet for our air pistol, it may be that not only air is caught behind the projectile, but that water may also become vaporized such that water vapour bubbles are formed behind the projectile. The effect, creating additional volume and leading to larger pressure, would be the same. This is also related to the general topic of cavitation.)

We note that a quantitative description of the processes within the interior of the egg or the water-filled sphere as well as directly behind the projectile are quite complex and would probably require numerical solutions of the Navier–Stokes equation with time resolution at least in the μs range.

Finally, the sequence of figure 18 also allows us to study the slowing down of the bullet for a much longer distance in water than with an egg or Christmas-tree ornament. And this offers another interesting physical phenomenon: the speed of the bullet is drastically reduced. This slowing down can be estimated from the well-known equation for the frictional force of bodies moving within fluids:

\[ F_R = \frac{1}{2} \rho A v^2 \]

where \( \rho \) denotes the density of the fluid, \( A \) the cross-sectional area of the moving object, \( v \) its velocity and \( c_w \) the drag coefficient (see e.g. [19]). The latter depends on the Rayleigh number, which itself depends on the viscosity and density of the liquid as well as the dimensions and velocity of the object that has an influence on the wake vortex. It may therefore change within the course of the experiment.

As an example, we calculate the drag force using the density of water (10³ kg m⁻³), the known area of a projectile (diameter 4.5 mm) and an average speed within the sphere of about 64 m s⁻¹ (average of 80 and 47 m s⁻¹). For a Reynolds number \( Re \) above \( 10^5 \) (as is the case in our experiment) we further arbitrarily assume a constant aerodynamic drag coefficient of around 0.5 (smaller \( Re \) leads to variation of \( c_w \) with \( Re \)). This gives an average drag force around 20 N. From Newton’s law, we calculated the velocity change \( \Delta v \) of the projectile within 1 ms—the time to traverse the diameter of the sphere—to be \( \Delta v = F \Delta t / m \), where \( m = 0.54 \) g, which gives about 40 m s⁻¹, which seems reasonably close to the observed 33 m s⁻¹. The respective deceleration amounts to about 40 000 m s⁻². This estimate could be improved by integrating the equation of motion; since, however, the input parameter \( c_w (v) \) is not accurately known, this makes no sense. In the experiment, the speed changed from around 54 m s⁻¹ after 54 mm water to less than 14 m s⁻¹ after having travelled four times this distance.

From figure 18 and the respective video (movie 11, available at stacks.iop.org/PhysED/47/664/mmedia), it can be seen that the projectile was so slow that it could not even penetrate the rubber skin. This feature helps to explain why it does indeed make sense in action or crime movies to jump into water and dive when someone is shooting: the frictional forces are quite large and lead to slowing down of projectiles across quite short distances.

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Oscillating droplets and incompressible liquids


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